Doctoral Abstract

Hybrid Information Theoretic Measures and Their Applications in Data Mining

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INTRODUCTION

Information theory, introduced by Shannon (1948), revolutionized the understanding of communication and uncertainty by providing a mathematical measure of information through entropy. Shannon entropy quantifies the average uncertainty associated with a random variable and has been widely applied in communication systems, statistics, pattern recognition, machine learning, and signal processing. Despite its success, Shannon's framework relies on precise probability distributions, which are often unavailable or unreliable in practical situations.

In real-life applications such as medical diagnosis, decision-making, pattern recognition, and data mining, information is frequently imprecise, incomplete, or vague. Human reasoning itself is not binary but involves partial truths and subjective judgments. Classical probabilistic models fail to adequately represent such complexities. To overcome these limitations, non-probabilistic approaches based on fuzzy set theory, rough set theory, soft set theory, and intuitionistic fuzzy set theory were developed.

Fuzzy set theory, introduced by Zadeh in 1965, allows elements to belong to a set with varying degrees of membership. Rough set theory, proposed by Pawlak in 1982, models uncertainty using lower and upper approximations. Soft set theory, introduced by Molodtsov in 1999, provides a parameterized framework for uncertainty representation. Intuitionistic fuzzy sets, developed by Atanassov in 1986, extend fuzzy sets by incorporating degrees of non-membership and hesitation.

This doctoral research integrates information theory with these uncertainty-handling frameworks to develop hybrid information theoretic measures. The primary motivation is to enhance the capability of entropy, similarity, and distance measures in handling complex and uncertain data. The focus is on applications in data mining and decision-making, where effective uncertainty modeling is crucial for accurate analysis and knowledge discovery.

REVIEW OF LITERATURE

The evolution of information theory represents a systematic effort to quantify uncertainty and information content in communication and decision-making systems. Early foundations of information theory can be traced to telecommunication studies by Hartley (1928) and Nyquist (1924, 1928), who examined signal transmission capacity and speed. These works laid the groundwork for the formal development of information theory by Shannon (1948), who introduced entropy as a quantitative measure of uncertainty in probabilistic experiments. Shannon's formulation revolutionized communication theory and subsequently influenced diverse disciplines such as physics, statistics, economics, biology, psychology, and computer science.

Following Shannon's work, probabilistic information measures were extensively generalized and applied. Weaver (1949) broadened the accessibility of Shannon's ideas, while Kullback (1959) introduced divergence measures to quantify differences between probability distributions. Rényi (1961) proposed a parametric generalization of Shannon entropy, allowing flexibility in sensitivity to probability distributions. Further extensions were developed by Havrda and Charvát (1967), Sharma and Taneja (1977), Tsallis (1988), and Pal and Pal (1989), leading to a wide class of generalized entropy measures. These probabilistic measures found successful applications in clustering, pattern recognition, image processing, and statistical inference. However, their reliance on precise probability distributions limited their effectiveness in real-world situations characterized by vagueness and incomplete information.

To address these limitations, Zadeh (1965) introduced fuzzy set theory, which allows partial membership of elements and provides a natural framework for modeling imprecision inherent in human reasoning. Zadeh (1968) further extended information theory by proposing fuzzy entropy, marking the beginning of non-probabilistic information measures. De Luca and Termini (1972) formalized axiomatic conditions for fuzzy entropy, establishing a foundation for subsequent developments. Later contributions by Kaufman (1975), Yager (1979), Kosko (1986), Pal and Pal (1992), and Bhandari and Pal (1993) introduced various fuzzy entropy measures based on distance, exponential forms, and divergence concepts. These measures demonstrated superior performance in applications such as medical diagnosis, pattern recognition, and decision-making.

The scope of uncertainty modeling expanded further with the introduction of rough set theory by Pawlak (1982), which addresses vagueness through lower and upper approximations without requiring additional information such as membership functions or probability distributions. Rough set theory gained prominence in feature selection, knowledge discovery, and data mining. The integration of fuzzy and rough set theories led to fuzzy rough sets, which combine granularity and fuzziness and have been shown to be particularly effective in attribute reduction and classification problems.

Atanassov (1986) proposed intuitionistic fuzzy sets, extending fuzzy sets by incorporating degrees of non-membership and hesitation. This additional parameter enabled more expressive modeling of uncertainty, especially in situations involving incomplete or conflicting information. Researchers such as Szmidt and Kacprzyk (2001), Wang and Xin (2005), and Hung and Yang (2008) developed distance, similarity, and entropy measures for intuitionistic fuzzy sets, further enhancing their applicability in decision-making and pattern recognition.

Soft set theory, introduced by Molodtsov (1999), provide a parameterized framework for uncertainty modeling, particularly useful in decision-making environments. Subsequent developments developments led to fuzzy soft sets and intuitionistic fuzzy soft sets, which integrate fuzziness and intuitionism with parameterization. Researchers including Maji et al. (2001), Majumdar and Samanta (2011), Jiang et al. (2013), and Li (2014) proposed various distance, similarity, and entropy measures within these frameworks, demonstrating applications in decision support systems and data analysis.

To capture higher levels of uncertainty, interval-valued fuzzy soft sets and interval-valued intuitionistic fuzzy soft sets were introduced. These models allow membership and non-membership values to be expressed as intervals, offering greater flexibility and realism. Contributions by Yang et al. (2009), Mukherjee and Sarkar (2014, 2015), Feng et al. (2017), and Sulaiman et al. (2018) advanced distance, similarity, and entropy measures in these settings, with applications in medical diagnosis and multi-criteria decision-making.

Despite the extensive literature on individual uncertainty models, existing studies reveal a research gap in the systematic development of hybrid information theoretic measures that unify fuzzy, rough, soft, intuitionistic, and interval-valued frameworks within a consistent information-theoretic perspective. Moreover, comparative studies highlighting the effectiveness of such hybrid measures in data mining tasks—such as feature selection, data reduction, and classification—remain limited. The present research addresses this gap by proposing novel hybrid entropy, distance, and similarity measures and demonstrating their applicability in data mining and decision-making under complex uncertainty environments.

OBJECTIVES OF THE STUDY

The primary objectives of the present doctoral research are:

1. To develop novel non-probabilistic and hybrid entropy measures for various uncertainty models.

2. To propose new distance and similarity

measures for fuzzy, intuitionistic, soft, and rough set-based structures.

3. To verify the theoretical validity and mathematical properties of the proposed measures.

4. To apply the developed measures to data mining tasks such as feature selection and data reduction.

5. To demonstrate applicability in decision-making and medical diagnosis problems.

6. To compare the proposed measures with existing methods and evaluate their effectiveness.

Proposed Measures

1. Trigonometric Entropy for Fuzzy Rough Set (FRS) and their Application in Medical Area

Some trigonometric information entropies have been proposed for FRSs and fuzzy rough values, and their validity is also proved. Corresponding to proposed trigonometric entropies, the weighted trigonometric entropies have been proposed for fuzzy rough sets.

Let A be FRS in $\Psi = \{\varphi_1, \varphi_2, \ldots \ldots, \varphi_n\}$, where $A = \sum_{i=1}^{n} \left[\langle \underline{\varphi_i}, \underline{\varphi_i} \rangle\right] / \varphi_i \; x_i \in \Psi$,

$$E_{sin}(A) = \frac{1}{n}\sum_{i=1}^{n} e_{A;\,sin}(\varphi_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} sin\; sin\left[\frac{\pi}{2}\left(1 - \frac{1}{2}\left(\left|2\underline{\varphi_i} - 1\right| + \left|2\underline{\varphi_i} - 1\right|\right)\right)\right], \forall \varphi_i \in \Psi, where\ A \in \Gamma^R(\Psi); \varphi_i = \langle \underline{\varphi_i}, \underline{\varphi_i} \rangle$$

$$E_{cos}(A) = \frac{1}{n}\sum_{i=1}^{n} e_{A;\,cos}(\varphi_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} cos\; cos\left[\frac{\pi}{4}\left(\left(\left|2\underline{\varphi_i} - 1\right| + \left|2\underline{\varphi_i} - 1\right|\right)\right)\right], \forall \varphi_i \in \Psi, where\ A \in \Gamma^R(\Psi); \varphi_i = \langle \underline{\varphi_i}, \underline{\varphi_i} \rangle$$

$$E_{tan}(A) = \frac{1}{n}\sum_{i=1}^{n} e_{A;\,tan}(\varphi_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} tan\; tan\left[\frac{\pi}{4}\left(1 - \frac{1}{2}\left(\left|2\underline{\varphi_i} - 1\right| + \left|2\underline{\varphi_i} - 1\right|\right)\right)\right], \forall \varphi_i \in \Psi, where\ A \in \Gamma^R(\Psi); \varphi_i = \langle \underline{\varphi_i}, \underline{\varphi_i} \rangle$$

then corresponding to equation (2.3.1), (2.3.2) and (2.3.3). Novel entropies are given below

The proposed framework integrates concepts from fuzzy sets and rough sets to better capture uncertainty and imprecision present in real-world datasets. By combining these paradigms, the measures aim to provide a more reliable representation of incomplete and vague information. The theoretical foundation of these measures is established through axiomatic

It is obvious that proposed entropies lie in the interval [0,1]. Larger the value of entropies the more is uncertainty in A.

After that an application of proposed measures is also discussed which are related to medical diagnosis and data reduction is also done in fuzzy rough environment.

Fuzzy rough set theory offers the flexibility to deal with two types of uncertainty present in information related to decision making or data related problems in daily life. It incorporates fuzzy set theory which considers vagueness within the rough set framework handling uncertain information. A variety of measures and methods for this integration have been proposed in the literature. In this thesis some trigonometric entropies were proposed for fuzzy rough environment and their validity is also proved. Finally, an application of these proposed measures has been used in medical diagnosis and data reduction problems. This shows the significance of proposed measures. These proposed measures can be used in other real-life problems for fuzzy rough environment.

2. Novel Distance Similarity and Entropy measures for intuitionistic fuzzy set (IFS)

Some new distance measures for IFSs are derived. Based on these distance measures, similarity measures are also derived for intuitionistic fuzzy sets. Distance and similarity measures are used to compare the differentiation and similarity respectively between two sets, two patterns, two images and two decisions etc. For comparing two intuitionistic fuzzy sets distance and similarity measures are studied by Szmidt and Kacprzyk

(2018), Luo (2018), Cheng et al. (2019).

Maximin distance measure

$$D_1(\Gamma,\Pi),(I,\Lambda)=\frac{1}{3}\left\{Min_j\left(\begin{matrix}|\Gamma\varphi_j-I\varphi_j|+\\|\Gamma^*\varphi_j-I^*\varphi_j|+|\Gamma^{**}\varphi_j-I^{**}\varphi_j|\end{matrix}\right)\right\}$$

Average distance measure

$$D_2(\Gamma,\Pi),(I,\Lambda)=\frac{1}{6n}\left\{\begin{matrix}\sum_{j=1}^{n}(|\Gamma\varphi_j-I\varphi_j|+|\Gamma^*\varphi_j-I^*\varphi_j|+|\Gamma^{**}\varphi_j-I^{**}\varphi_j|)\\+Max_j\{(|\Gamma\varphi_j-I\varphi_j|+|\Gamma^*\varphi_j-I^*\varphi_j|+|\Gamma^{**}\varphi_j-I^{**}\varphi_j|+)\}\end{matrix}\right\}$$

Convex distance measure

$$D_3(\Gamma,\Pi),(I,\Lambda)=\frac{1}{3}\left\{\alpha Max_j(|\Gamma\varphi_j-I\varphi_j|+|\Gamma^*\varphi_j-I^*\varphi_j|+|\Gamma^{**}\varphi_j-I^{**}\varphi_j|)+(1-\alpha)\left\{Min_j\left(\left(\begin{matrix}|\Gamma\varphi_j-I\varphi_j|+|\Gamma^*\varphi_j-I^*\varphi_j|+\\+|\Gamma^{**}\varphi_j-I^{**}\varphi_j|\end{matrix}\right)\right)\right\}\right\}$$

Similarity measures for IFS

Distance and similarity are dual to each other. Larger the distance is smaller the similarity between any two sets. This concept is used to define below some similarity measures based on above defined distance measures. Among similarity measures proposed by other researchers, some of those, however, did not satisfy the axioms of similarity or provide counterintuitive cases or are produced in complex way. To overcome this drawback some new similarity measures are derived. The proposed similarity measure depends on the tuples (membership degree, non-membership degree, and dubious factor). Szmidt E, Kacprzyk J (2004), Ye (2011), Shi and Ye (2013), Tian (2013), Rajarajeswari and Uma (2013), Papakostas et al., (2013), have done lot of work in the field of similarity measure of IFS.

Maximin similarity measure:

$$S_1(\Gamma,\Pi),(I,\Lambda)=1-\frac{1}{3}\left\{Min_j\left(\begin{matrix}|\Gamma\varphi_j-I\varphi_j|+\\|\Gamma^*\varphi_j-I^*\varphi_j|\\+|\Gamma^{**}\varphi_j-I^{**}\varphi_j|\end{matrix}\right)\right\}$$

Average similarity measure:

$$S_2(\Gamma,\Pi),(I,\Lambda)=1-\frac{1}{6}\left\{\begin{matrix}\sum_{j=1}^{n}\frac{(|\Gamma\varphi_j-I\varphi_j|+|\Gamma^*\varphi_j-I^*\varphi_j|+|\Gamma^{**}\varphi_j-I^{**}\varphi_j|)}{n}\\+Max_j\{(|\Gamma\varphi_j-I\varphi_j|+|\Gamma^*\varphi_j-I^*\varphi_j|++|\Gamma^{**}\varphi_j-I^{**}\varphi_j|+)\}\end{matrix}\right\}$$

Convex similarity measure is presented as:

$$S_3(\Gamma,\Pi),(I,\Lambda)=1-\frac{1}{3}\{\alpha Max_j(|\Gamma\varphi_j-I\varphi_j|+|\Gamma^*\varphi_j-I^*\varphi_j|+|\Gamma^{**}\varphi_j-I^{**}\varphi_j|)+(1-\alpha)\{Min_j((|\Gamma\varphi_j-I\varphi_j|+|\Gamma^*\varphi_j-I^*\varphi_j|++|\Gamma^{**}\varphi_j-I^{**}\varphi_j|))\}\}$$

This study applies intuitionistic fuzzy set (IFS)–based similarity measures to support rational and informed career determination for students by incorporating membership, non-membership, and hesitation degrees derived from academic performance across key subjects. Using maximin, average, and convex similarity measures, the similarity between four students and four potential career options—medicine, pharmacy, surgery, and anatomy—is systematically evaluated. The results consistently indicate that surgery emerges as the most suitable career choice for most students across all similarity measures, highlighting its robustness as a preferred option. While the maximin and average similarity measures show stable outcomes with minor variations, the convex similarity measure demonstrates sensitivity to the parameter α, revealing how changes in decision-maker emphasis influence career recommendations. Notably, at lower α values, anatomy and medicine occasionally become competitive alternatives, particularly for specific students, whereas higher α values reinforce surgery as the dominant choice. Overall, the analysis confirms that IFS-based similarity measures effectively capture uncertainty and partial knowledge in student performance, offering a flexible and reliable quantitative framework for career guidance and decision-making.

3. Distance, Similarity and Entropy measures for fuzzy soft set (FSS)

It discusses distance, similarity, and entropy measures for fuzzy soft set. Some distance measures are derived by using the concept of average, maximin and convexity by taking the modulus values of two sets. Further, some similarity measures are also proposed as these measures are useful to check whether the two sets are significantly similar. Under fuzzy soft settings, some entropy measures are also proposed using similarity measures. Finally, the proposed similarity measure is applied to a real-world decision-making problem.

Distance measures of FSS

The following distance measures are proposed for two fuzzy soft sets (Γ,Π) and (I,Λ)

Maximin distance measure:

$$D_1\big((\Gamma,\Pi),(I,\Lambda)\big) = Max_i\{Min_j(|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|)\} \ldots (4.3.1)$$

Average distance measure:

$$D_2\big((\Gamma,\Pi),(I,\Lambda)\big) = \frac{1}{2m}\sum_{i=1}^{m}\left(\sum_{j=1}^{n}\left(\frac{|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|}{n}\right) + Max(|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|)\right)$$

Convex distance measure:

$$D_3\big((\Gamma,\Pi),(I,\Lambda)\big) = Max_i\left\{\begin{matrix}\alpha Max_j(|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|)\\ +(1-\alpha)Min_j(|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|)\end{matrix}\right\}$$

Maximin similarity measure is presented as:

$$S_1\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - Max_i\{Min_j(|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|)\} \ldots (4.3.3)$$

Average similarity measure is presented as:

$$S_2\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - \frac{1}{2m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\begin{matrix}\left(\frac{|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|}{n}\right)\\ +Max(|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|)\end{matrix}\right\}$$

Convex Similarity Measure is presented as:

$$S_3\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - Max_i\{\alpha Max_j(|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|) + (1-\alpha)Min_j(|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j|)\}$$

Entropy measures for FSS

Let consider (Γ,Π) is a fuzzy soft set for all and $\varphi_j \in \Phi$, i=1,2…m and j=1,2…n then

Entropy based on Maximin Similarity Measure is presented as:

$$E_1(\Gamma,\Pi) = S_1\big((\Gamma,\Pi),(\Gamma^c,\Pi)\big) = 1 - Max_i\{Min_j(|2\Gamma_{\theta_i}\varphi_j - 1|)\}$$

Entropy based on Average second Similarity Measure is presented as:

$$E_2(\Gamma,\Pi) = S_2\big((\Gamma,\Pi),(\Gamma^c,\Pi)\big) = 1 - \frac{1}{2m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{|\Gamma_{\theta_i}\varphi_j - \Gamma^c{}_{\theta_i}\varphi_j|}{n} + Max(|\Gamma_{\theta_i}\varphi_j - \Gamma^c{}_{\theta_i}\varphi_j|)\right\}$$

Entropy based on Convex Similarity Measure is presented as:

$$E_3(\Gamma,\Pi) = S_3\big((\Gamma,\Pi),(\Gamma^c,\Pi)\big) = 1 - Max_i\{\alpha Max_j(|\Gamma_{\theta_i}\varphi_j - \Gamma^c{}_{\theta_i}\varphi_j|) + (1-\alpha)Min_j(|\Gamma_{\theta_i}\varphi_j - \Gamma^c{}_{\theta_i}\varphi_j|)\}$$

The thesis highlights the application of fuzzy soft entropy and similarity measures as effective tools for decision-making under uncertainty. By modeling real-life problems such as investment selection and student award allocation, fuzzy soft sets capture the vagueness inherent in human judgment and multi-criteria evaluation. Entropy measures are used to assess the amount of information available for making reliable decisions, while similarity measures help compare alternatives against an ideal or reference model. The results demonstrate that these measures provide consistent and rational rankings, supporting optimal decision outcomes. Overall, the study confirms that fuzzy soft set–based entropy and similarity frameworks are robust, flexible, and well-suited for complex decision-making environments where precise information is difficult to obtain.

4. Distance, Similarity and Entropy measures for intuitionistic fuzzy soft set (IFSS)

Several fields dealing with the problem of uncertainty do not get fruitful results using fuzzy set theory, rough set theory, probability theory and other mathematical approaches. Then Soft set (SS) theory has been derived by Russian researcher Molodtsov (1999) that becomes very helpful to deal with the issues of uncertainty and vagueness. Further by taking hybridization of IFS and FSS distance, similarity and entropy measures are derived for IFSS. Firstly, distance measures are proposed by using the concept of average, maximin, convexity after then corresponding to proposed distance measures similarity measures are also derived. Based on these similarity measures, entropy measures are described for the intuitionistic fuzzy soft sets. Then a decision-making problem is also discussed by using entropy and similarity measures numerically.

Distance measures for IFSS

The following distance measures are proposed for two fuzzy soft sets (Γ,Π) and (I,Λ)

Maximin distance measure

$$D_1\big((\Gamma,\Pi),(I,\Lambda)\big) = Max_i\left\{Min_j\left(\frac{\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|}{2}\right)\right\}$$

Average distance measure

$$D_2\big((\Gamma,\Pi),(I,\Lambda)\big) = \frac{1}{4m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|}{n} + Max\left(\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|\right)\right\}$$

Convex distance measure

$$D_3\big((\Gamma,\Pi),(I,\Lambda)\big) = Max_i\left\{\alpha Max_j\left(\frac{\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|}{2}\right) + (1-\alpha)Min_j\left(\frac{\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|}{2}\right)\right\}$$

Similarity measures for IFSS

Maximin similarity measure

$$S_1\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - \frac{1}{2}Max_i\left\{Min_j\left(\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|\right)\right\}$$

Average similarity measure

$$S_2\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - \frac{1}{4m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|}{n} + Max\left(\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|\right)\right\}$$

Convex similarity measure

$$S_3\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - \frac{1}{2}Max_i\left\{\alpha Max_j\left(\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|\right) + (1-\alpha)Min_j\left(\left|\Gamma_{\theta_i}\varphi_j - I_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - I^*{}_{\theta_i}\varphi_j\right|\right)\right\}$$

Entropy measures for IFSS

Let consider (Γ,Π) is a fuzzy soft set for all and **φj** **Φ**, i=1,2…m and j=1,2…n then:

Maximin entropy measure

$$E_1\big((\Gamma,\Pi),(\Gamma^c,\Pi)\big) = 1 - \left(\frac{1}{2}\right)Max_i\left\{Min_j\left(\left|\Gamma_{\theta_i}\varphi_j - \Gamma^c{}_{\theta_i}\varphi_j\right| + \left|\Gamma^*{}_{\theta_i}\varphi_j - \Gamma^{*c}{}_{\theta_i}\varphi_j\right|\right)\right\}$$

$$= 1 - \left(\frac{1}{2}\right)Max_i\left\{Min_j\left(\left|2\Gamma_{\theta_i}\varphi_j - 1\right| + \left|2\Gamma^*{}_{\theta_i}\varphi_j - 1\right|\right)\right\}$$

Average entropy is presented as:

$$E_2((\Gamma,\Pi),(\Gamma^c,\Pi)) = 1 - \frac{1}{4m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{|2\Gamma_{\theta_i}\varphi_j - 1| + |2\Gamma^*{}_{\theta_i}\varphi_j - 1|}{n} + Max(|2\Gamma_{\theta_i}\varphi_j - 1| + |2\Gamma^*{}_{\theta_i}\varphi_j - 1|)\right\}$$

Convex entropy is presented as:

$$E_3((\Gamma,\Pi),(\Gamma^c,\Pi)) = 1 - \left(\frac{1}{2}\right) Max_i\{\alpha Max_j(|2\Gamma_{\theta_i}\varphi_j - 1| + |2\Gamma^*{}_{\theta_i}\varphi_j - 1|) + (1 - \alpha)Min_j(|2\Gamma_{\theta_i}\varphi_j - 1| + |2\Gamma^*{}_{\theta_i}\varphi_j - 1|)\}$$

Intuitionistic fuzzy soft entropy measures play an important role in handling uncertainty and hesitation in real-world decision-making environments. In the context of market prediction, these measures effectively capture the attractiveness of different companies by simultaneously considering membership and non-membership information under multiple decision criteria. This approach enables investment organizations to evaluate alternatives in a structured and reliable manner, even when expert opinions and market conditions are imprecise or incomplete. The entropy- based analysis demonstrates that the available information is sufficient and consistent, thereby supporting confident investment decisions.

Similarly, intuitionistic fuzzy soft similarity measures provide a systematic framework for decision-making in personnel selection. When evaluating candidates foran information technology company, multiple qualitative attributes such as qualifications, skills, experience, and personality are assessed using intuitionistic fuzzy information obtained from different judges. By comparing these evaluations with an established standard derived from past experience, similarity measures help identify the most suitable candidates. The results indicate that the evaluations provided by one judge exhibit a higher degree of similarity to the standard, suggesting a better alignment with organizational expectations.

Overall, the intuitionistic fuzzy soft set framework proves to be highly effective for addressing complex decision-making problems characterized by vagueness and uncertainty. The development of distance, similarity, and entropy measures enhances the analytical capability of this framework. The applications discussed illustrate that intuitionistic fuzzy soft methods offer reliable and flexible tools for market analysis and human resource selection, thereby contributing significantly to informed and rational decision-making processes.

5. Distance, Similarity and Entropy measures for Interval-Valued Fuzzy Soft Set

Most of the time in real-life problems the membership value in FS is not significant or relevant. It is better to present interval-valued data to describe the membership degree of such sets. Yang et al. (2009) introduced the concept of IVFSS by combining IVFS and SS models. Recently, Yiara yong (2020) presented the idea of combining the theories of interval-valued FSS over semigroups. Alkhazaleh and Salleh (2012) discussed the idea of generalized IVFSS and their properties The problems in decision making area were also solved simultaneously by using similarity and distance measures for IVFSS by them. Alkhazaleh and Salleh (2011) gave the significant prospective soft set to intuitionistic FSS.

Distance measures for IVFSS

The following distance measures are proposed for two fuzzy soft sets (Γ,Π) and (I,Λ)

Maximin distance measure:

$$D_1((\Gamma,\Pi),(I,\Lambda)) = Max_i\left\{Min_j\left(\frac{|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|}{2}\right)\right\}$$

Average distance measure:

$$D_2((\Gamma,\Pi),(I,\Lambda)) = \frac{1}{4m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|}{n} + Max\left(|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|\right)\right\}$$

Convex distance measure:

$$D_3((\Gamma,\Pi),(I,\Lambda)) = Max_i\left\{\alpha Max_j\left(\frac{\left(|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|\right)}{2}\right) + (1-\alpha)Min_j\left(\frac{\left(|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|\right)}{2}\right)\right\}$$

Similarity Measures for IVFSS

Maximin similarity measure:

$$S_1((\Gamma,\Pi),(I,\Lambda)) = 1 - Max_i\left\{Min_j\left(\frac{|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|}{2}\right)\right\}$$

Average similarity measure:

$$S_2((\Gamma,\Pi),(I,\Lambda)) = 1 - \frac{1}{4m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|}{n} + Max\left(|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|\right)\right\}$$

Convex similarity measure:

$$S_3((\Gamma,\Pi),(I,\Lambda)) = 1 - Max_i\left\{\alpha Max_j\left(\frac{\left(|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|\right)}{2}\right) + (1-\alpha)Min_j\left(\frac{\left(|\Gamma^l{}_{\theta_i}\varphi_j - I^l{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - I^u{}_{\theta_i}\varphi_j|\right)}{2}\right)\right\}$$

Entropy measures for IVFSS

Maximin entropy is presented as:

$$E_1((\Gamma,\Pi),(\Gamma^c,\Pi)) = 1 - \left(\frac{1}{2}\right)Max_i\{Min_j(|\Gamma^l{}_{\theta_i}\varphi_j - \Gamma^{c\,l}{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - \Gamma^{c\,u}{}_{\theta_i}\varphi_j|)\} = 1 - \left(\frac{1}{2}\right)Max_i\{Min_j(|2\Gamma^l{}_{\theta_i}\varphi_j - 1| + |2\Gamma^u{}_{\theta_i}\varphi_j - 1|)\}$$

Average entropy is presented as:

$$E_2((\Gamma,\Pi),(\Gamma^c,\Pi)) = 1 - \frac{1}{4m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{|2\Gamma^l{}_{\theta_i}\varphi_j - 1| + |2\Gamma^u{}_{\theta_i}\varphi_j - 1|}{n} + Max(|2\Gamma^l{}_{\theta_i}\varphi_j - 1| + |2\Gamma^u{}_{\theta_i}\varphi_j - 1|)\right\}$$

Convex entropy is presented as:

$$E_3((\Gamma,\Pi),(\Gamma^c,\Pi)) = 1 - \left(\frac{1}{2}\right)Max_i\{\alpha Max_j(|\Gamma^l{}_{\theta_i}\varphi_j - \Gamma^{c\,l}{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - \Gamma^{c\,u}{}_{\theta_i}\varphi_j|) + (1-\alpha)Min_j(|\Gamma^l{}_{\theta_i}\varphi_j - \Gamma^{c\,l}{}_{\theta_i}\varphi_j| + |\Gamma^u{}_{\theta_i}\varphi_j - \Gamma^{c\,u}{}_{\theta_i}\varphi_j|)\}$$

$$= 1 - \left(\frac{1}{2}\right)Max_i\{\alpha Max_j(|2\Gamma^l{}_{\theta_i}\varphi_j - 1| + |2\Gamma^u{}_{\theta_i}\varphi_j - 1|) + (1-\alpha)Min_j(|2\Gamma^l{}_{\theta_i}\varphi_j - 1| + |2\Gamma^u{}_{\theta_i}\varphi_j - 1|)\}$$

Interval-valued fuzzy soft set theory provides an effective framework for handling uncertainty and vagueness in emergency decision-making situations. In the case of a coal mine explosion accident, several emergency response plans are evaluated using benefit-based parameters such as gas concentration control, casualty reduction, smoke and dust management, feasibility of rescue operations, and restoration of damaged facilities. By representing expert assessments through interval-valued fuzzy information, the proposed entropy measures successfully quantify the amount of useful information contained in the data. The analysis shows that certain entropy measures yield higher informational content, while others contribute comparatively less, indicating their relative effectiveness in decision support.

Similarly, interval-valued fuzzy soft distance measures are applied to assess the severity of impact in regions affected by a tropical storm. Relief measures related to agriculture, livestock, fishermen, loss of life, alternative livelihood support, and ecological damage are evaluated using historical records and expert opinions. By comparing these assessments through distance measures, the approach clearly distinguishes between areas with different levels of impact. The results consistently indicate that one region is more severely affected and requires greater relief intervention than the other. This conclusion is further supported by existing normalized distance measures from the literature, confirming the reliability and robustness of the proposed approach.

Overall, decision-making under uncertain and complex conditions is significantly strengthened with interval-valued fuzzy soft entropy, distance, and similarity measures. These tools provide a systematic and consistent means of evaluating alternatives and identifying priorities in emergency management scenarios. The proposed measures can be further extended to develop additional information measures and may also be adapted to other hybrid soft set models, thereby offering wide applicability in future research and real-world decision-making problems.

6. Distance, Similarity and Entropy measures for Interval-Valued Fuzzy Soft Set

In this section by taking hybridization of IFS and IVFSS three different distance measures are defined along with their properties. Further based on these measures three similarity and entropy measures are also derived. Thereafter a hypothetical data is used to demonstrate applicability of proposed entropy measure in decision making. Finally, to check the validity of proposed distance measures, these measures are applied to the area of medical diagnosis.

Distance measures for IVIFSS

The following distance measures are proposed for two fuzzy soft sets (Γ,Π) and (I,Λ)

Maximin Distance Measure

$$D_1\big((\Gamma,\Pi),(I,\Lambda)\big) = \frac{1}{6} Max_i\Big\{Min_j\Big(\left|\Gamma^{l}{}_{\theta_i}\varphi_j - I^{l}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{u}{}_{\theta_i}\varphi_j - I^{u}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{l^*}{}_{\theta_i}\varphi_j - I^{l^*}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{u^*}{}_{\theta_i}\varphi_j - I^{u^*}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{l^{**}}{}_{\theta_i}\varphi_j - I^{l^{**}}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{u^{**}}{}_{\theta_i}\varphi_j - I^{u^{**}}{}_{\theta_i}\varphi_j\right|\Big)\Big\}$$

Average Distance Measure

$$D_3\big((\Gamma,\Pi),(I,\Lambda)\big) = Max_i\left\{\alpha Max_j\left(\frac{\left(\begin{array}{c}\left|\Gamma^{l}{}_{\theta_i}\varphi_j - I^{l}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{u}{}_{\theta_i}\varphi_j - I^{u}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{l^*}{}_{\theta_i}\varphi_j - I^{l^*}{}_{\theta_i}\varphi_j\right| \\ + \left|\Gamma^{u^*}{}_{\theta_i}\varphi_j - I^{u^*}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{l^{**}}{}_{\theta_i}\varphi_j - I^{l^{**}}{}_{\theta_i}\varphi_j\right| \\ + \left|\Gamma^{u^{**}}{}_{\theta_i}\varphi_j - I^{u^{**}}{}_{\theta_i}\varphi_j\right|\end{array}\right)}{6}\right) + (1-\alpha) Min_j\left(\frac{\left(\begin{array}{c}\left|\Gamma^{l}{}_{\theta_i}\varphi_j - I^{l}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{u}{}_{\theta_i}\varphi_j - I^{u}{}_{\theta_i}\varphi_j\right| \\ + \left|\Gamma^{l^*}{}_{\theta_i}\varphi_j - I^{l^*}{}_{\theta_i}\varphi_j\right| + \left|\Gamma^{u^*}{}_{\theta_i}\varphi_j - I^{u^*}{}_{\theta_i}\varphi_j\right| \\ + \left|\Gamma^{l^{**}}{}_{\theta_i}\varphi_j - I^{l^{**}}{}_{\theta_i}\varphi_j\right| \\ + \left|\Gamma^{u^{**}}{}_{\theta_i}\varphi_j - I^{u^{**}}{}_{\theta_i}\varphi_j\right|\end{array}\right)}{6}\right)\right\}$$

Convex Distance Measure

$$D_3\big((\Gamma,\Pi),(I,\Lambda)\big) = Max_i\left\{\alpha Max_j\left(\frac{\begin{pmatrix}|\Gamma^{l}{}_{\theta_i\varphi_j} - I^{l}{}_{\theta_i\varphi_j}| + |\Gamma^{u}{}_{\theta_i\varphi_j} - I^{u}{}_{\theta_i\varphi_j}| + |\Gamma^{l^*}{}_{\theta_i\varphi_j} - I^{l^*}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{u^*}{}_{\theta_i\varphi_j} - I^{u^*}{}_{\theta_i\varphi_j}| + |\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - I^{l^{**}}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - I^{u^{**}}{}_{\theta_i\varphi_j}|\end{pmatrix}}{6}\right) + (1-\alpha) Min_j\left(\frac{\begin{pmatrix}|\Gamma^{l}{}_{\theta_i\varphi_j} - I^{l}{}_{\theta_i\varphi_j}| + |\Gamma^{u}{}_{\theta_i\varphi_j} - I^{u}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{l^*}{}_{\theta_i\varphi_j} - I^{l^*}{}_{\theta_i\varphi_j}| + |\Gamma^{u^*}{}_{\theta_i\varphi_j} - I^{u^*}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - I^{l^{**}}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - I^{u^{**}}{}_{\theta_i\varphi_j}|\end{pmatrix}}{6}\right)\right\}$$

Similarity measures for IVIFSS

Maximin similarity measure :

$$S_1\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - \frac{1}{6} Max_i\left\{Min_j\begin{pmatrix}|\Gamma^{l}{}_{\theta_i\varphi_j} - I^{l}{}_{\theta_i\varphi_j}| + |\Gamma^{u}{}_{\theta_i\varphi_j} - I^{u}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{l^*}{}_{\theta_i\varphi_j} - I^{l^*}{}_{\theta_i\varphi_j}| + |\Gamma^{u^*}{}_{\theta_i\varphi_j} - I^{u^*}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - I^{l^{**}}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - I^{u^{**}}{}_{\theta_i\varphi_j}|\end{pmatrix}\right\}$$

Average similarity measure:

$$S_2\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - \frac{1}{12m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{\begin{matrix}|\Gamma^{l}{}_{\theta_i\varphi_j} - I^{l}{}_{\theta_i\varphi_j}| + |\Gamma^{u}{}_{\theta_i\varphi_j} - I^{u}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{l^*}{}_{\theta_i\varphi_j} - I^{l^*}{}_{\theta_i\varphi_j}| + |\Gamma^{u^*}{}_{\theta_i\varphi_j} - I^{u^*}{}_{\theta_i\varphi_j}| + |\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - I^{l^{**}}{}_{\theta_i\varphi_j}| + |\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - I^{u^{**}}{}_{\theta_i\varphi_j}|\end{matrix}}{n} + Max\big(|\Gamma^{l}{}_{\theta_i\varphi_j} - I^{l}{}_{\theta_i\varphi_j}| + |\Gamma^{u}{}_{\theta_i\varphi_j} - I^{u}{}_{\theta_i\varphi_j}| + |\Gamma^{l^*}{}_{\theta_i\varphi_j} - I^{l^*}{}_{\theta_i\varphi_j}| + |\Gamma^{u^*}{}_{\theta_i\varphi_j} - I^{u^*}{}_{\theta_i\varphi_j}| + |\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - I^{l^{**}}{}_{\theta_i\varphi_j}| + |\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - I^{u^{**}}{}_{\theta_i\varphi_j}|\big)\right\}$$

Convex similarity measure

$$S_3\big((\Gamma,\Pi),(I,\Lambda)\big) = 1 - Max_i\left\{\alpha Max_j\left(\frac{\begin{pmatrix}|\Gamma^{l}{}_{\theta_i\varphi_j} - I^{l}{}_{\theta_i\varphi_j}| + |\Gamma^{u}{}_{\theta_i\varphi_j} - I^{u}{}_{\theta_i\varphi_j}| + |\Gamma^{l^*}{}_{\theta_i\varphi_j} - I^{l^*}{}_{\theta_i\varphi_j}| \\ + |\Gamma^{u^*}{}_{\theta_i\varphi_j} - I^{u^*}{}_{\theta_i\varphi_j}| + |\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - I^{l^{**}}{}_{\theta_i\varphi_j}| + |\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - I^{u^{**}}{}_{\theta_i\varphi_j}|\end{pmatrix}}{6}\right) + (1-\alpha) Min_j\left(\frac{\big(|\Gamma^{l}{}_{\theta_i\varphi_j} - I^{l}{}_{\theta_i\varphi_j}| + |\Gamma^{u}{}_{\theta_i\varphi_j} - I^{u}{}_{\theta_i\varphi_j}| + |\Gamma^{l^*}{}_{\theta_i\varphi_j} - I^{l^*}{}_{\theta_i\varphi_j}| + |\Gamma^{u^*}{}_{\theta_i\varphi_j} - I^{u^*}{}_{\theta_i\varphi_j}| + |\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - I^{l^{**}}{}_{\theta_i\varphi_j}| + |\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - I^{u^{**}}{}_{\theta_i\varphi_j}|\big)}{6}\right)\right\}$$

Entropy measures for IVIFSS

Maximin Entropy Measure

$$E_1\big((\Gamma,\Pi),(\Gamma^c,\Lambda)\big) = 1 - \left(\frac{1}{6}\right) Max_i\Big\{Min_j\big(|2\Gamma^{l}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{l^*}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u^*}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - 1|\big)\Big\}$$

Average entropy measures

$$E_2\big((\Gamma,\Pi),(\Gamma^c,\Lambda)\big) = 1 - \frac{1}{12m}\sum_{i=1}^{m}\left\{\sum_{j=1}^{n}\frac{\begin{matrix}|2\Gamma^{l}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u}{}_{\theta_i\varphi_j} - 1| \\ + |2\Gamma^{l^*}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u^*}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - 1|\end{matrix}}{n} + Max\big(|2\Gamma^{l}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{l^*}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u^*}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{l^{**}}{}_{\theta_i\varphi_j} - 1| + |2\Gamma^{u^{**}}{}_{\theta_i\varphi_j} - 1|\big)\right\}$$

Convex entropy measure

$$E_3\left((\Gamma,\Pi),(\Gamma^c,\Lambda)\right) = 1 - \frac{1}{6} Max_i \left\{ \alpha Max_j \left(\begin{array}{c} \left|2\Gamma^l{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^u{}_{\theta_i\varphi_j}-1\right| \\ + \left|2\Gamma^{*l}{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^{*u}{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^{l^{**}}{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^{u^{**}}{}_{\theta_i\varphi_j}-1\right| \end{array} \right) + (1-\alpha) Min_j \left(\left|2\Gamma^l{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^u{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^{*l}{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^{*u}{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^{l^{**}}{}_{\theta_i\varphi_j}-1\right| + \left|2\Gamma^{u^{**}}{}_{\theta_i\varphi_j}-1\right| \right) \right\}$$

Interval-valued intuitionistic fuzzy soft entropy measures provide an effective mechanism for decision-making problems in situations where information is imprecise and evaluations are expressed in ranges rather than exact values. In the context of house selection, the attractiveness of different houses is assessed using qualitative parameters such as cost, aesthetics, material quality, maintenance condition, and surrounding environment. Since exact evaluations are difficult to obtain, interval-valued intuitionistic fuzzy soft sets successfully capture the uncertainty associated with human judgment. The entropy analysis reveals that certain entropy measures extract more informative content from the data, while others contribute comparatively less, thereby highlighting their relative effectiveness in decision support.

The applicability of interval-valued intuitionistic fuzzy soft distance measures is further demonstrated in the field of medical diagnosis. Here, patient symptoms are compared with predefined diagnostic patterns representing different diseases. By calculating distances between the patient profile and diagnostic profiles, the proposed measures assist in identifying the most plausible diagnosis. The results show that different distance measures may lead to different diagnostic conclusions, offering a broader perspective for clinical decision-making. In contrast, existing normalized distance measures from the literature provide limited discrimination, whereas the proposed measures offer clearer and more reliable insights.

Overall, interval-valued intuitionistic fuzzy soft set theory represents a powerful integration of interval-valued intuitionistic fuzzy sets and soft set theory. The development of distance, similarity, and entropy measures within this framework significantly enhances its analytical capability. The applications presented illustrate the robustness and flexibility of the proposed measures in decision-making and medical diagnosis under uncertainty. These methods can be further extended to develop additional information measures and adapted to other hybrid soft set models, thereby opening new directions for future research.

CONCLUSION

The present doctoral research has made a significant contribution to the domain of information theory by systematically developing and analyzing hybrid information theoretic measures capable of handling multiple forms of uncertainty encountered in real-world data. Traditional probabilistic information measures, though mathematically elegant and widely applicable, rely heavily on precise probability distributions and therefore fall short when confronted with vagueness, ambiguity, incompleteness, and subjectivity. This research successfully addresses these limitations by integrating information theory with fuzzy sets, rough sets, soft sets, intuitionistic fuzzy sets, and their interval-valued and hybrid extensions.

A comprehensive framework for non-probabilistic and hybrid entropy, distance, and similarity measures has been proposed. The theoretical foundations of these measures were rigorously established through axiomatic definitions and mathematical proofs, ensuring essential properties such as non-negativity, boundedness, symmetry, monotonicity, and consistency. The inclusion of hesitation, boundary regions, and parameterization significantly enhances the expressive power of the proposed measures compared to classical significantly enhances the expressive power of the proposed measures compared to classical counterparts.

The applicability and effectiveness of the develop measures were demonstrated through their use in data mining tasks, including feature selection, attribute reduction, and pattern classification, as well as in decision-making and medical diagnosis problems. The results indicate that hybrid information theoretic measures provide improved robustness, better discrimination capability, and higher interpretability when dealing with uncertain and imprecise data. In particular, the integration of fuzzy, rough, and soft computing paradigms enables a more realistic representation of real-world knowledge systems.

Overall, the research confirms that hybrid information theoretic approaches constitute a powerful and flexible alternative to probabilistic methods, especially in complex environments where uncertainty cannot be adequately captured by probability alone. The outcomes of this study not only enrich the theoretical literature but also offer practical tools for researchers and practitioners working in data analytics, artificial intelligence, and decision support systems.

FUTURE SCOPE

While the present research provides a strong theoretical and application-oriented foundation, several promising directions remain open for further investigation:

1. Extension to Dynamic and Time-Dependent Data: The proposed measures can be extended to dynamic environments where data evolve over time. Developing entropy and similarity measures for temporal fuzzy, rough, or intuitionistic frameworks would be particularly useful for real-time decision-making, financial forecasting, and monitoring systems.

2. Integration with Machine Learning and Deep Learning Models: Future research may explore the incorporation of hybrid information theoretic measures into machine learning algorithms such as clustering, classification, ensemble learning, and deep neural networks. These measures can be used as loss functions, feature selection criteria, or similarity metrics to enhance model interpretability and robustness.

3. Big Data and High-Dimensional Applications: With the increasing availability of large-scale and high-dimensional datasets, there is scope to adapt the proposed measures for big data environments. Optimization technique sand parallel computing strategies may be employed to improve computational efficiency and scalability.

4. Development of Decision Support Systems: The hybrid measures can be embedded into intelligent decision support systems for applications in healthcare, supply chain management, risk assessment, and policy analysis. User-friendly software tools and visualization techniques may further enhance practical usability.

5. Uncertainty Modelling in Emerging Domains: Emerging fields such as Internet of Things (IoT), cyber-physical systems, social network analysis, and smart cities involve heterogeneous and uncertain data sources. The proposed framework can be extended to model uncertainty in these complex systems.

6. Further Theoretical Generalizations: Future work may focus on developing new axiomatic systems and generalized hybrid frameworks by combining intuitionistic, neutrosophic, and plithogenic sets with information theory. Such extensions could provide even richer representations of indeterminacy and contradiction.

7. Real Data: Future research may focus on systematic primary real data collection through surveys, expert evaluations, field observations, and institutional records to validate and refine hybrid information theoretic measures, ensuring their robustness, adaptability, and practical relevance under real-world uncertainty and incomplete information scenarios.

REFERENCES

i. Alkhazaleh, S., & Salleh, A. R. (2012). Generalised interval-valued fuzzy soft set. Journal of Applied Mathematics, Article ID 870504.

ii. Alkhazaleh, S., Salleh, A. R., & Hassan, N. (2011). Fuzzy parameterized interval-valued fuzzy soft set. Applied Mathematical Sciences, 5(67), 3335–3346.

iii. Bhandari, D., & Pal, N. R. (1993). Some new information measures for fuzzy sets. Information Sciences, 67, 209–228.

iv. Cheng, C., Xiao, F., & Cao, Z. (2019). A new distance for intuitionistic fuzzy sets based on similarity matrix. IEEE Access, 7, 70436–70446.

v. De Luca, A., & Termini, S. (1972). A definition of a non-probabilistic entropy in the setting of fuzzy sets. Information and Control, 20, 301–312.

vi. Du, W. S., & Hu, B. Q. (2015). Aggregation distance measure and its induced similarity measure between intuitionistic fuzzy sets. Pattern Recognition Letters, 60, 65–71.

vii. Feng, Q., & Guo, X. (2017). Uncertainty measures of interval-valued intuitionistic fuzzy soft sets and their applications in decision making. Intelligent Data Analysis, 21(1), 77–95.

viii. Hartley, R. V. L. (1928). Transmission of information. Bell System Technical Journal, 7, 535–563.

ix. Havrda, J. H., & Charvát, F. (1967). Quantification method of classification processes: Concept of structural α-entropy. Kybernetika, 3, 30–35.

x. Hung, W. L., & Yang, M. S. (2006). Fuzzy entropy on intuitionistic fuzzy sets. International Journal of Intelligent Systems, 21(4), 443–451.

xi. Jiang, Y. C., Tang, Y., Liu, H., & Chen, Z. Z. (2013). Entropy on intuitionistic fuzzy soft sets and on interval-valued fuzzy soft sets. Information Sciences, 240, 95–114.

xii. Kaufmann, A. (1975). Introduction to the theory of fuzzy subsets: Fundamental theoretical elements. Academic Press.

xiii. Kullback, S. (1959). Information theory and statistics. John Wiley & Sons.

xiv. Li, Y., Qin, K., & He, X. (2014). Some new approaches to constructing similarity measures. Fuzzy Sets and Systems, 234, 46–60.

xv. Luo, M., & Zhao, R. (2018). A distance measure between intuitionistic fuzzy sets and its application in medical diagnosis. Artificial Intelligence in Medicine, 89, 34–39.

xvi. Luo, X., Li, W., & Zhao, W. (2018). Intuitive distance for intuitionistic fuzzy sets with applications in pattern recognition. Applied Intelligence, 48, 2792–2808.

xvii. Mahanta, J., & Panda, S. (2020). A novel distance measure for intuitionistic fuzzy sets with diverse applications. International Journal of Intelligent Systems. https://doi.org/10.1002/int.22312

xviii. Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets. Journal of Fuzzy Mathematics, 9(3), 589–602.

xix. Molodtsov, D. (1999). Soft set theory—First results. Computers & Mathematics with Applications, 37(4–5), 19–31.

xx. Mukherjee, A., & Sarkar, S. (2014). Similarity measures for interval-valued intuitionistic fuzzy soft sets and its application in medical diagnosis problem. New Trends in Mathematical Sciences, 2(3), 159–165.

xxi. Mukherjee, A., & Sarkar, S. (2015). Distance-based similarity measures for interval-valued intuitionistic fuzzy soft sets and its application. New Trends in Mathematical Sciences, 3(4), 34–42.

xxii. Pal, N. R., & Pal, S. K. (1989). Object background segmentation using new definitions of entropy. IEEE Proceedings, 366, 284–295.

xxiii. Papakostas, G. A., Hatzimichailidis, A. G., & Kaburlasos, V. G. (2013). Distance and similarity measures between intuitionistic fuzzy sets: A comparative analysis from a pattern recognition point of view. Pattern Recognition Letters, 34(14), 1609–1622.

xxiv. Pawlak, Z. (1982). Rough sets. International Journal of Information and Computer Sciences, 11, 341–356.

xxv. Rajarajeswari, P., & Uma, N. (2013). Intuitionistic fuzzy multi similarity measure based on cotangent function. International Journal of Engineering Research & Technology, 2(11), 1323–1329.

xxvi. Rényi, A. (1961). On measures of entropy and information. In Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability (Vol. 1, pp. 547–562). University of California Press.

xxvii. Shannon, C. E. (1948). A mathematical theory of communication. Bell System Technical Journal, 27, 379–423.

xxviii. Sharma, B. D., & Taneja, I. J. (1977). Three generalized additive measures of entropy. Ekonomicko-Matematicky Obzor, 13, 419–433.

xxix. Shi, L. L., & Ye, J. (2013). Study on fault diagnosis of turbine using an improved cosine similarity measure for vague sets. Journal of Applied Sciences, 13(10), 1781–1786.

xxx. Szmidt, E., & Kacprzyk, J. (2000). Distances between intuitionistic fuzzy sets. Fuzzy Sets and Systems, 114(3), 505–518.

xxxi. Szmidt, E., & Kacprzyk, J. (2001). Entropy for intuitionistic fuzzy sets. Fuzzy Sets and Systems, 118(3), 467–477.

xxxii. Szmidt, E., & Kacprzyk, J. (2004). A similarity measure for intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning. In Lecture Notes in Computer Science (Vol. 3070). Springer.

xxxiii. Tian, M. Y. (2013). A new fuzzy similarity based on cotangent function for medical diagnosis. Advanced Modeling and Optimization, 15(2), 151–156.

xxxiv. Tsallis, C. (1988). Possible generalization of Boltzmann–Gibbs statistics. Journal of Statistical Physics, 52, 479–487.

xxxv. Wang, W., & Xin, X. (2005). Distance measure between intuitionistic fuzzy sets. Pattern Recognition Letters, 26, 2063–2069.

xxxvi. Weaver, W. (1949). The mathematics of communication. Scientific American, 181, 11–15.

xxxvii. Xu, Z. (2007). Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making. Fuzzy Optimization and Decision Making, 6(2), 109–121.

xxxviii. Yager, R. R. (1979). On the measure of fuzziness and negation. Part I: Membership in the unit interval. International Journal of General Systems, 5(4), 221–229.

xxxix. Yang, X. B., Lin, T. Y., Yang, J. Y., Li, Y., & Yu, D. (2009). Combination of interval-valued fuzzy set and soft set. Computers & Mathematics with Applications, 58(3), 521–527.

xl. Ye, J. (2011). Cosine similarity measures for intuitionistic fuzzy sets and their applications. Mathematical and Computer Modelling, 53, 91–97.

xli. Yiarayong, P. (2020). On interval-valued fuzzy soft set theory applied to semigroups. Soft Computing, 24(8), 3113–3168.

xlii. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8, 338–353.

xliii. Zadeh, L. A. (1968). Probability measures of fuzzy events. Journal of Mathematical Analysis and Applications, 23, 421–427.