An Optimization Approach To Fueling Warships At Sea

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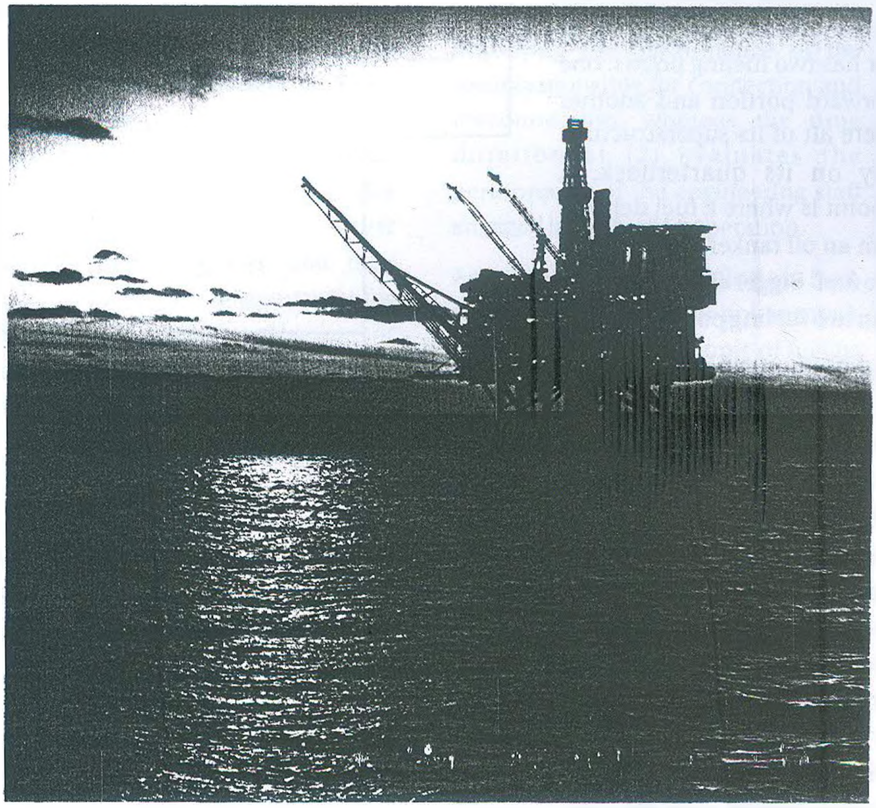
BSTRACT

eling at sea is an essential and vital eration for warships. Not only ring wartime but also when ships nduct exercises during peacetime, plenishment at sea is carried out to nsfer men and material from one p to another. Fueling is a subset of replenishment at sea operations. he fueling operation is not carried carefully, either a fuel spillage urs which is a fire hazard and sumes many man-hours in ning the mess, or the fueling ation increases that makes the ship and the oil tanker a good et for the enemy. In this paper, the ical scenario of fueling at sea is cussed. A mathematical gramming model is formulated solved using the optimal control ry. This model is based on the ior's experience in fueling der class frigates, a type of hip used by many countries. e rules of thumb are devised to ate the pumping rate in the ng process. The rules would tate the ship's Engineer Officer nderstand the control action out getting involved in the ematical jargon.

WORDS: Fueling at sea, nishment at sea, ematical Model, Optimal ol Theory



INTRODUCTION

Warships conduct exercises at sea on a regular basis. There are two main objectives for conducting war exercises: first to keep various systems on board in operational condition, and secondly to impart training to new members of the ship's crew. A fleet is comprised of many ships, generally of different functional utilities, such as an aircraft carrier, anti-submarine ships, frigates, destroyers, mine sweepers, an oil tanker, and a cruiser etc. Ships of one or more fleets go together for conducting war exercises in an open sea and may stay there for weeks.

An important and inevitable part of an exercise for warships is "replenishment at sea." During this operation, men and materials are transferred from one ship to another while ships move at 15-20 knots. Fueling at sea is concurrently carried out with the "replenishment at sea" operation. During the operation, fuel is pumped to a warship from an oil tanker, also a member of the warship fleet. In general, an oil tanker is capable of fueling three ships simultaneously; one on its port side, another on its starboard side and the third at its astern. A frigate or a destroyer has two fueling points: one on its forward portion and another somewhere aft of its superstructure, generally on its quarterdeck. A fueling point is where a fuel delivery hose from an oil tanker is connected to receive fuel. Bigger ships may have more than two fueling points.

The objective of this paper is to analyze the fueling operation of warships at sea and suggest an optimal operating procedure or a set of guidelines. We formulate a mathematical programming model of the fueling process and then solve it. The model provides an insight into the fueling operation. The suggested procedure will help the ship's staff carry out fueling more efficiently and economically. The next section describes the fueling process. The section on problem formulation describes the mathematical model. A solution to the problem is obtained and explained thereafter.

PROBLEM DESCRIPTION

During the replenishment at sea, two or more ships move together at the same speed of fifteen to twenty knots, at a distance of eighty to one hundred feet, and on the same course. Replenishment at sea is a very well planned activity. Each member of the team involved in the activity has to be meticulous at his task. At the predetermined time, the warship to be fueled approaches the oil tanker of the fleet on its starboard or port side, becomes parallel to the tanker, and adjusts its speed and course to that of the oil tanker. After the two ships have stabilized in their speed and the course, the tanker shoots a thin rope to the warship and a designated member of the crew pulls the rope.

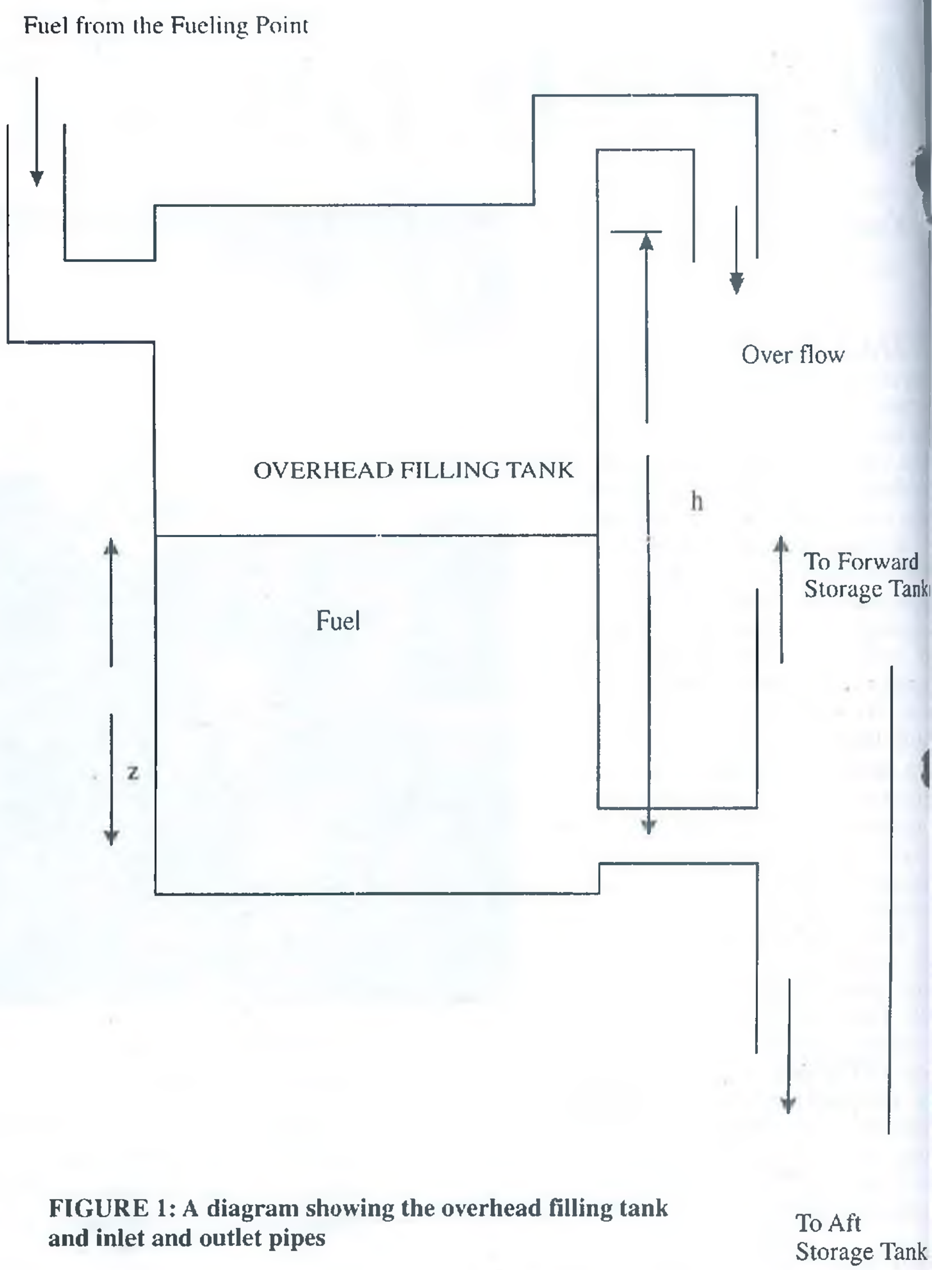


FIGURE 1: A diagram showing the overhead filling tank and inlet and outlet pipes

rope gradually becomes thicker, eventually, the fuel delivery hose ched to the rope is passed on to warship. The hose is then nected to the warship's fueling it.

a "start pumping" signal from the ship, the tanker starts pumping which flows to an overhead g tank through a pipeline from fueling point. At its bottom, the g tank has an outlet pipe that rcates into two branches: one s to the forward storage tanks and ther to the aft storage tanks, both d in the double bottom of the . At the bifurcation point, there is hange over valve that lets the fuel either to the forward tanks or to aft tanks. Fuel flows by gravity m the overhead-filling tank to the rage tanks as shown in Figure 1.
eship's Engineer Officer, in charge the fueling operation, stays at the ding point during the fueling cess, monitors and controls the pumping rate. He has a direct phone line connected to the ker, and another connected to a tion near the filling tank. nicians are placed at appropriate s to open or close the valves to s fuel storage tanks. While ing, it is required to maintain the 's stability. To do so, fuel valves are operated as to let the fuel flow to e than one tank simultaneously, on the port side and the other on starboard side. The state of the tanks and the level of fuel in the g tank is continuously monitored informed to the Engineer Officer. ending on the state of fuel storage s and the overhead-filling tank, warship Engineer Officer municates with the Engineer Officer of the tanker either to vary the pumping rate or to stop pumping.

The essential point to note here is that the flow of fuel, to and from the filling tank, is required to be monitored and regulated continuously, mainly, to avoid the overflow of the filing tank. When the filling tank becomes full, it overflows on to the ship's main deck or to its sides. Fuel spillage is considered to be highly undesirable. It is a great fire hazard. And secondly, it takes many man-hours to clean spillage from the deck and sides. If the Engineer Officer is extra cautious and maintains a low level in the filling tank, the fueling time increases unnecessarily and excessively. Since the two ships, the tanker and the warship, move at a constant speed on a constant course during the replenishment operation, their maneuverability is lost, and they become a good and bigger target for the enemy. Therefore, replenishment at sea is ought to be performed in the minimum possible time.

When the required quantity of fuel has been received, pumping is stopped. Then, the fuel delivery hose is blown through with air to empty it out. It is disconnected from the fuel tank; its open mouth is blocked with a steel plate to avoid any spillage in case traces of fuel are still left in the hose, and the hose is pulled back. During the replenishment operation, fresh water and/or men and material can also be transferred from one ship to another while fueling. If no other replenishment is going on, then the warship moves away from the tanker and another warship approaches for fueling and replenishment.

The fleet commander records the following three time durations during the replenishment operation:

1. The time a warship becomes parallel to the tanker to the time the delivery hose is connected at the fueling point and the tanker is informed to start pumping.
2. The time taken from "start pumping" to "stop pumping" i.e.. the fueling duration. This helps compute the average fueling rate.
3. The time taken from "stop pumping" to the final disconnection of the warship from the tanker.

The time durations at (1) and (3) above are helpful in evaluating and improving the performance of the team responsible for connection and disconnection, whereas the time duration at (2) evaluates the performance of the engineering staff engaged in the fueling operation.

An attempt was made to find out from open literature whether any research has been done on the topic of fueling at sea. Breickner (1962) describes the progress made over the last decade in replenishment techniques, but does not mention anything about the fueling process. Dankers and Huntley (1963) describe logistics development and include the equipment used in replenishment at sea. Morisseau (1972) discusses about future developments in making replenishment ships. These articles are on replenishment-at-sea in Naval Engineers Journal, but none were found on fueling at sea, addressing the problem described above. These articles include mainly the progress made in transferring material and ammunition to warships at sea, and how rigs, various components, and ships used for replenishment evolved.

Jaconson, Lee and Speyer (1971) derive necessary conditions for optimality. Maurer (1977) and McIntyre and Paiewonsky (1967) describe solution methods for solving optimal control problems with bounded state variables. Sethi and Thompson (1981) present various applications and scenarios of optimal control theory, formulate and solve them.

PROBLEM FORMULATION

It is now obvious from the scenario described above that the goal is to minimize the fueling duration, or to maximize the quantity of fuel received on board a warship during the pumping time, T. The constraint is that the filling tank must not overflow. This problem is formulated as a mathematical model using optimal control theory. In the model, the fuel pumping rate is a control variable and the fuel level in the filling tank is a state variable.

The Parameters are defined as follows:

A = the cross-sectional area of the filling tank.

d = the diameter of the outlet pipe from the filling tank.

a = the cross-sectional area of the outlet pipe from the filling tank, where $a = \pi d^2/4$.

g = acceleration due to gravity = 9.806754 m/sec^2

h = the height of the filling tank beyond which spillage occurs.

The variables are:

p (t) = the pumping rate at time t. It is the control variable.

z (t) = the fuel level in the filling tank at time t. It is the state variable.

z (t) = z(t)/dt = the rate at which the fuel level varies in the filling tank.

Using the variables and parameters, the following expressions can be stated:

Rate of fuel in-flow to the filling tank at time t = p(t),

Rate of fuel out-flow from the filling tank at time t = $a\,(2\,g\,z(t))^{0.5}$

The net rate of flow into the filling tank at time t is:

$p(t) - a\,(2\,g\,z(t))^{0.5}$

The above equations are based on the assumption that the velocity of fuel on the surface of the filling tank is practically zero.

Now, the objective function is:

$$\text{Maximize} \quad \{ J = \int_0^T p(t)\,dt \} \quad \ldots \quad (1)$$

Subject to

$$\dot{z} = \{1/A\}\{p(t) - a\,\text{Sqrt}(2\,g\,z(t))\}, \quad z(0) = 0 \quad \ldots \quad (2)$$

$$z(t) \le h \quad \ldots \quad (3)$$

$$0 \le p(t) \le p_{max} \quad \ldots \quad (4)$$

Here, the interpretation of the objective function (1) is that we are trying to find the trajectory of the state variable z(t) so that the area under the curve p(t) is maximized. This also means that the quantity of fuel received in time T is maximized. Or, time T is minimized for a given quantity of fuel received. The state equation (2) expresses the rate at which the fuel level in the filling tank varies, with the initial condition of no fuel in the filling tank at the start of the fueling operation. Inequality (3) restrict the fuel level to at most h, the critical level, beyond which spillage occurs. The conditions at (4) restricts the control variable to be non-negative and less than or equal to the maximum pumping rate. This problem has the Lagrange form.

SOLUTION TO THE PROBLEM

While solving the problem, z may be used instead of z(t) and p, instead of p(t). Let λ be the adjoint variable.

The hamiltonian is:

$$H = p + \{\lambda/A\}\{p(t) - a\,\text{Sqrt}(2\,gz(t))\} \quad \ldots \quad (5)$$

Equation (5) is linear in p. Therefore, the first derivative of the Hemiltonion,

$$H_p = 1 + \lambda/A \quad \ldots \quad (6)$$

For such systems, $H_p = 0$ implies that the coefficient of the linear control term vanishes identically along a singular arc according to Sethi and Thompson (1981). The control is not determined in terms of x and λ by the Hamiltonian maximizing condition $H_p=0$. Instead, the control is determined by the requirement that the coefficients of these terms remain zero on the singular arc. Therefore, the time derivative of H_p must be zero. Having obtained this condition (or setting higher time derivatives equal to zero) along the singular arc, an additional necessary condition analogous to the concavity condition needs to be checked. For a maximizing problem with a single control variable, the generalized Legendre Clebsch condition is:

$(-1)^k \partial/\partial \; p[(d/dt)^{2k} \; H_p] \leq 0, \quad k=0,1,2,$ .... (7)

In order to get an adjoint equation and multipliers associated with the constraints, a Lagrangian function should be formed. Let μ be the multiplier associated with constraint (3). Then, the Lagrangian is stated as follows:

$L = H + \mu\,(h-x)$ .... (8)

Or $L = p + \{\lambda/A\}\{p(t) - a\,\mathrm{Sqrt}\,(2\,g\,z(t))\} + \mu\,(h-x)$ .. (9)

From the Lagrangian, the joint equation is obtained as follows:

$D\lambda/dt = -\partial L/\partial z = \mu + (a\lambda/A)(\mathrm{Sqrt}\,(2\,g))\,(0.5\,z^{-.5})$

$= \mu + (a\lambda/2A)(\mathrm{Sqrt}\,(2\,g/z)), \; \lambda(T) = O$ .... (10)

and μ must also satisfy the following complementary slackness conditions:

$h-z \geq 0, \qquad \mu \geq 0$, and $\qquad \mu(h-z) = 0$ .... (11)

Based on the above equations and conditions, we can state that the optimal control is bang-bang plus singular. Let $p^*(t)$ be the optimal pumping rate, then

$p^*(t) = \mathrm{bang}\,(0, P_{max}; 1 + \lambda/A)$ .... (12)

And the singular arcs must satisfy the following equation:

$H_p = 1 + \lambda/A = O$ .... (13)

Now, the optimal control along the singular arc can be obtained by:

$d/dt\,H_p = d\lambda/dt = 0$

Or

$d\lambda/dt = \mu + (a\lambda/2A)(\mathrm{Sqrt}\,(2g/z)) = 0$ .... (14)

Differentiating once more with respect to time t, we obtain:

$d^2/dt^2(H_p) = (-a\lambda/4A)(\mathrm{Sqrt}\,(2\,g))(z^{-1.5})\,(dz/dt)$ .... (15)

Or

$(-a\lambda/4A)(\mathrm{Sqrt}\,(2\,g))\,(z^{-1.5})(p - a\sqrt{2\,g\,z}) = 0$ .... (16)

Equation (16) implies that along the singular arc either

$\{p - a\,\mathrm{Sqrt}\,(2\,g\,z)\} = 0$

or $z = 0$ .... (17)

The derivative of the left hand side of (16) with respect to p also satisfies the generalized Legendre Clebsch condition.

The adjoint equation (10) involves z. and the state equation (2) involves p, and p depends on λ. Therefore, initially we cannot solve equation (10) until the differential equation (2) is solved and equation (2) cannot be integrated either. The way out of this dilemma is to use some intuition. Our goal is to maximize the objective function value $J = \int p(t)\,dt$, and it is equivalent to maximizing the total area under the curve p(t). This area can be maximized only if the pumping rate is initially increased to p_{max}. Therefore, $1 + \lambda/A$ must be positive for $z \leq h$. And when $z = h$, the singular control ought to be applied. With this assumption, we solve the differential equation for z.

$\dot{z} = p/A - (a/A)\,(\mathrm{Sqrt}\,(2\,g\,z)), \; z(0) = 0$ .... (18)

Let $y = z^{.5}$, then $\dot{y} = dy/dt$

$\dot{y} = (0.5\,z^{-.5})(\dot{z})$

Or

$2\,y\,\dot{y} = \dot{z}$ .... (19)

Substituting the value of $z^{.5}$ and $\dot{z}$, we get

$2\,y\,\dot{y} = p/A - (a/A)\,(\mathrm{Sqrt}\,(2\,g))(y)$,

And after algebraic manipulation,

$\dot{y} = \{p/A - (a/A)\,(\mathrm{Sqrt}\,(2\,g))(y)\}/2y$

Or

$dy/dt = \{p/A - (a/A)\,(\mathrm{Sqrt}\,(2\,g))(y)\}/2y$

Or

$y\,dy/\{p/A - (a/A)\,(\mathrm{Sqrt}\,(2\,g))$

$(y)\} = 0.5\,dt$ .... (20)

Integrating both sides,

$\int y\,dy/\,/\{p/A - (a/A)\,(\mathrm{Sqrt}\,(2\,g))$

$(y)\} = \int 0.5\,dt$ .... (21)

We know that

$\int y\,dy\,/(a + by) = y/b - a\,\mathrm{Ln}(a + b\,z)\,/\,b^2$ .... (22)

Applying this rule, the solution to (21) is as follows:

$Ay\,/\,a(\mathrm{Sqrt}\,(2\,g)) - \{A\,p\,/\,2\,a^2\,g\}\mathrm{Ln}\,\{p/A - (a/A)\,(\mathrm{Sqrt}\,(2\,g))(y)\} = 0.5\,t + c$ .... (23)

Here, c is the constant of integration. On substituting y for $z^{0.5}$, we get the following expression:

$A\,z^{0.5}\,/\,a(\sqrt{2\,g} - \{A\,p\,/\,2\,a^2\,g\}\mathrm{Ln}\,\{p/A - (a/A)\,(\sqrt{2\,g\,z})\}$

$= 0.5\,t + c$ .... (24)

Since $z(0) = 0$, the constant of integration c is as follows:

$c = (A\,p/\,2\,a^2\,g)\,\mathrm{Ln}\,(p/A)$ .... (25)

Substituting the value of constant c, we get

$-A\,z^{0.5}\,/\,a(\mathrm{Sqrt}\,(2\,g) - \{A\,p\,/\,2\,a^2\,g\}\mathrm{Ln}\,\{p/A - (a/A)\,(\mathrm{Sqrt}\,(2\,g\,z))\}$

$= 0.5\,t - (A\,p/\,2\,a^2\,g)^*\,\mathrm{Ln}\,(p/A)\,g)^*\,\mathrm{Ln}\,(p/A)$

After simplifying the above equation, the following expression is obtained:

$(a/p)\,\mathrm{Sqrt}\,(2\,g\,z) + \mathrm{Ln}\{\,1 - (a/p)\,\mathrm{Sqrt}\,(2\,g\,z)\}$

$= (a^2\,g\,/\,A\,p)\,t$ .... (26)

Equation (26) does not yield a close form solution of z. To obtain z(t), a trial and error method need to be used. If we know the value of the parameters, then z(t) can easily be computed using Microsoft Excel. Its special in-built program, Goal Seek, can easily compute the height of fuel in the tank, z, for different values of time, t.

Since $1 + \lambda / A$ is positive when the level of the filling tank, z, is less than h, the pumping rate at the beginning can be increased to the maximum possible value that the pipeline or the fuel hose can hold. Subsequently, when the level of the filling tank approaches h, a singular control needs to be applied.

Along the singular arc, it has been determined in equation (17) that p - a Sqrt (2gh) = 0

Or

p = a Sqrt (2gh) .... (27)

Equation (27) would be helpful in calculating the pumping rate when the fuel level approaches the critical point, i.e., the height of the overhead-filling tank.

COMPUTATIONAL EXPERIENCE

The cross-sectional area of the filling tank, A, is assumed to be 10 square meters and the diameter of the outlet pipe, d, is assumed to be 0.1 meters. For pumping rates of 300 to 600 cubic meters, the level of fuel in the filling tank, z, is computed for the first ten minutes using equation (26) and plotted against time. These calculations are listed in Table 1. In order to evaluate the effect of the outlet pipe diameter, the calculations are repeated for d = 0.15 meters, and are placed in Table 2. For various pumping rates, fuel levels in the filling tank are plotted against time and the graphs are placed at Figures 1 and 2 (See in Next Page).

SIMPLE RULES FOR THE SHIP'S ENGINEER OFFICER

In the previous sections, a mathematical programming model is formulated and its solution is obtained. However, Engineer Officers of warships may not like to get involved in all the jargons used above. Therefore, from the foregoing, the following thumb rules for the ship's Engineer Officer can be stated:

1. After the fuel hose is connected at the ship's fueling point, instruct the oil tanker to start pumping at the maximum rate.
2. Monitor the fuel level in the overhead filling tank continuously.
3. When the filling tank level approaches the top of the filling tank, instruct the oil tanker to reduce the pumping rate in accordance with Equation (27).

That is, $p = a\sqrt{2gh}$.

Table 1: Fuel level in the filling tank

Pipe Diameter, d = 10 centimeters
Filling Tank Cross-Sectional Area, A = 10 square meters

Time t in Minutes	Fuel Level in the Filling Tank at the Following Pumping Rate 300	400	500	600
0		0.000		0.000
1		0.560		0.868
2		1.041		1.635
3		1.473		2.340
4		1.869		2.999
5		2.237		3.620
6		2.580		4.206
7		2.901		4.764
8		3.202		5.296
9		3.487		5.804
10		3.757		6.291

Table 2: Fuel level in the filling tank

Pipe Diameter, d = 15 centimeters
Filling Tank Cross-Sectional Area, A = 10 square meters

Time t in Minutes	Fuel Level in the Filling Tank at the Following Pumping Rate 300	400	500	600
0	0.000	0.000	0.000	0.000
1	0.314	0.447	0.583	0.722
2	0.511	0.748	0.997	1.254
3	0.652	0.978	1.324	1.684
4	0.757	1.158	1.590	2.042
5	0.837	1.304	1.810	2.346
6	0.899	1.421	1.995	2.606
7	0.947	1.518	2.152	2.831
8	0.985	1.598	2.285	3.027
9	1.016	1.665	2.399	3.198
10	1.039	1.720	2.498	3.348

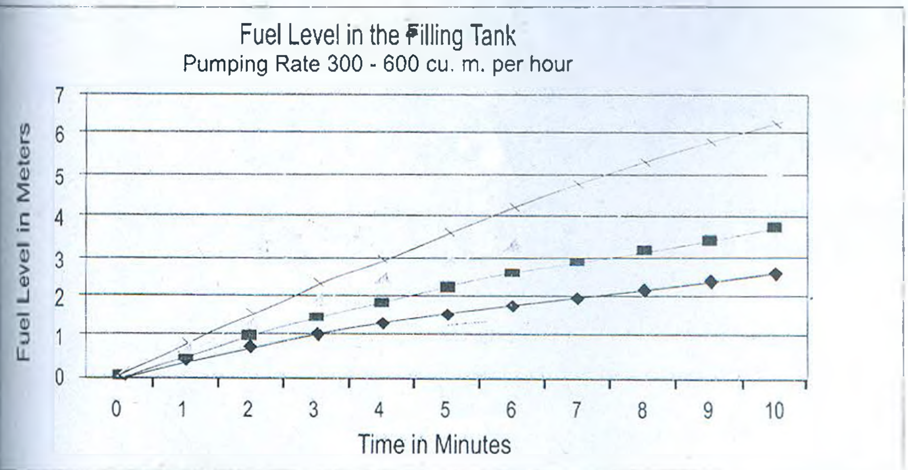


Figure 2: Graph Showing Fuel Level in Filling nk for outlet pipe diameter of 10 centimeters.

Continue with the pumping operation until the quired amount of fuel has been received on board, and op pumping.

he pumping rate in 3 above is calculated on the sumption that there is no pipe friction and there is a nooth flow of fuel from the filling tank to various storage nks. In actual practice this is not true. Pipes have different rvatures and joints that cause resistance and slow down e flow of fuel through the pipes. In addition to this type of sistance, the length of the pipeline from the filling tank to e storage tank plays an important role and further duces th rate of flow. Also, the specific gravity of the fuel s its effect on the flow rate. The specific gravity of a liquid dependent on its temperature. Therefore, in actual ctice the rate of flow throu h the pipeline is less than {a Sqrt(2 g h). Let us assume a flow factor (always less than 1). The ship's Engineer Officer can estimate the correct value of the flow factor because different ships may have different flow factors even though they are of the same class. Therefore, Equation (27) should be modified to read as:

p = Flow Factor {a Sqrt (2 g h)}

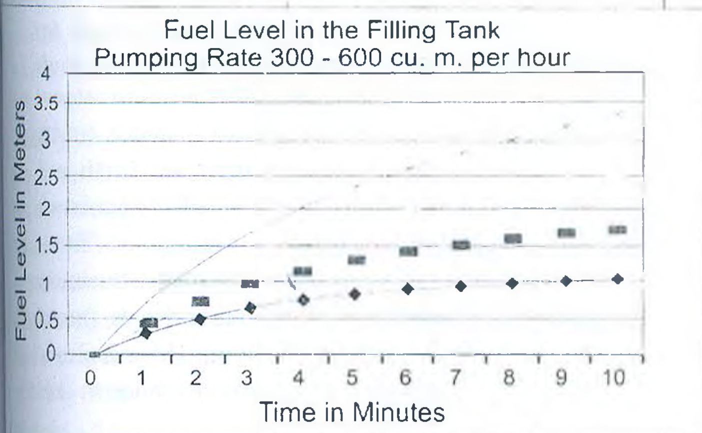


Figure 3: Graph Showing Fuel Level in Filling Tank for outlet pipe diameter of 15 centimeters.

CONCLUSION

In this paper, a real-world scenario is described and formulated as a mathematical model. It is a basic model and several other complexities are not included in it. The final outcome of the analysis presented in this paper appears to be intuitive, but this realization is only after the analysis is done. The author was an Engineer Officer in the Indian Navy and fueled Leander Class frigates and other types of warships at sea. The model and the thumb rules derived from its solution will benefit engineer officers of warships. Operating procedures for fueling at sea can be formulated based on this research.

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