ABSTRACT

Image registration is mapping positions in one image with the corresponding positions in the other image. Rigid registration of images using hierarchical decomposition is useful because a coarse to fine strategy helps in correctly estimating the mapping function parameters as we move to a finer resolution at each level. This also reduces search space and computational time. Multi resolution nature of wavelets is employed for image decomposition. The decomposed images are then registered using maximization of Mutual Information (MI). In this paper variation of MI and signal to noise ratio (SNR) with increase in level of decomposition and with use of Sub-sampling/filtering and wavelet decomposition has been analyzed. Results with wavelet decomposition have been found to be better than those with sub-sampling/filtering as with each reduced search space finer details are being analyzed rather than the entire image content and this is the inherent feature of wavelet decomposition.

Mutual Information Based *Image Registration:*

INTRODUCTION

Image registration is required in image analysis applications such as remote sensing, medical diagnosis disease monitoring etc. where information so obtained from multi modality images or from same modality at different times or from different view points is complementary in nature. Therefore it becomes necessary to integrate these images. The spatial alignment of these images is called image registration. This alignment requires application of a geometrical transformation like rigid, affine, curved [Maintz,Viergever 1998] transformations. The main issues involved in image registration are registration accuracy and time required for registration. Image registration involves suitably transforming the test image using the transformations to bring it close to the reference image, identifying features in test image to be compared using a suitable similarity measure and optimizing the similarity measure.

Transformation of images is required to relate points in one image to their corresponding points in the other image. Thus the registration classification is based on choice of suitable feature space, search space, search strategy and suitable similarity measure [Brown 1992]. Registration can be based on different similarity measures such as sum of squared distances, correlation ratio, correlation coefficient, etc. Mutual Information has been recognized as a powerful similarity measure. [Crum et al. 2004].In this paper, search space is confined to low frequency sub bands which helps in reducing the amount of computational complexity. Images to be registered are decomposed into low frequency sub bands by Discrete wavelet transform method and Sub-sampling/filtering method. It is observed that with different levels of decomposition images that are decomposed using Sub-sampling/ filtering and Discrete wavelet transform, Mutual Information (MI) is seen to vary. It also explains the effect of level of decomposition and the type of decomposition used on SNR of registered image.

An approach Using Wavelet Decomposition

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ROLE OF MUTUAL INFORMATION IN IMAGE REGISTRATION

Information is associated with the uncertainty about the occurrence of an event. Statistical parameter expressing this information is the entropy, which is defined as the average information per individual message in communication theory. In case of images, individual message is the information about gray level of the pixels forming the image. Entropy is thus a measure of uncertainty, which predicts the gray values of an image given the probability density function (pdf) of the gray values. Thus Mutual Information (MI) gives the amount of information one image contains about the other and therefore measures the gray value dependence between the images. Two images are said to be registered or matched if their Mutual Information is maximum.

So the advantage of using MI is that it does not assume any functional relationship between intensities of images, rather it depends on the statistical relationship between images. It is thus an intensity based approach which does not require any pre-processing or segmentation as with point based or surface based measures. Thus it reduces the processing complexity and is especially applied to multi resolution matching to improve the speed of matching.

Statistical Relations of Mutual Information

Mutual Information (I) of two images image A and image B is defined as

$$I(A,B) = H(A) + H(B) - H(A,B)$$
$$= H(A) - H(AB)$$
$$= H(B) - H(B/A)$$

where H(A) and H(B) are the entropies of images A and B respectively and H(A,B) is the joint entropy of images A and B. Entropy H(A) or H(B) is the shannon entropy given as

$$H(A) = -\Sigma_{a \in A}\, p_A(a) \log p_A(a)$$
$$H(B) = -\Sigma_{b \in B}\, p_B(b) \log p_B(b)$$
$$I(A,B) = \Sigma_{a,b}\, p_{AB}(a,b) \log p_{AB}(a,b) / p_A(a)\, p_B(b)$$
$$H(A,B) = -\Sigma_{a,b}\, p_{AB}(a,b) \log p_{AB}(a,b)$$

Joint entropy H(A,B) is computed from the joint probability pAB (a,b). pAB (a,b) is the co-occurrence of gray value b in image B or a in image A, at the same image position, for all a and b in the overlapping part of A and B. Joint probability is computed by initially finding a 2D joint histogram of the gray value combination (A, T(B)) for every grid point b of test image B that lies in the region of overlap of A and B. T(B) is the transform of image B. When the image is transformed, the transformed position of pixel/voxel does not coincide with the pixel/voxel of the original image. The histogram entry of image gray values which are to be increased are determined by interpolation. Partial volume interpolation is preferred to Nearest Neighbour and Trilinear interpolation because it considerably reduces interpolation artifacts as compared to other methods.

Joint histogram is computed from the overlap of test image and interpolated reference image [Hua-Mei, Varshney 2003] using a co-occurrence matrix, Cr(z,y) for certain relation r. Co-occurrence matrix represents an estimate of the probability that if a pixel (i1,j1) has intensity z and a pixel (i2,j2) has intensity y the information about the relation r is recorded in the square co-occurrence matrix Cr [Sonka, Hilavac, Boyle 2001] whose dimensions correspond to the number of brightness levels of the image.

Once the joint histogram using the co-occurrence matrix is found, it's histogram entries are divided by the total number of entries to create a probability density function (pdf). From these pdf's Marginal pdf's are found for A and B respectively for finding entropies H(A), H(B) and H(A,B) respectively. These are calculated by summing up the rows or columns of the joint histogram.

If two images are identical, the joint histogram is a diagonal. Among the other methods to determine joint entropy, Parzen Window estimate has also been used [Thevenaz, Unser 2000]. It uses the procedure of getting a test image on which the geometric transformation is applied and a reference image which will be interpolated. The intensity values in the reference image at every transformed grid point of floating image are estimated using intensity interpolation. The joint histogram of test image and interpolated reference image is then determined. Then mutual information is found using joint and probability mass functions of test and reference images as explained above. In our experiments we have used normalized mutual information as

$I(A,B) = (H(A) + H(B)) / H(A,B)$ [Hill, Batchelor et al. 2001].

INTERPOLATION

Interpolation as applied to images is finding the image intensity values at inter pixel positions. This is required because the pixel positions change in the transformed image .The idea here is to make use of neighbouring pixel values to determine intensity information at any pixel location. Three important interpolation methods are Nearest Neighbour, Trilinear and Partial Volume Interpolation.

Nearest Neighbour Interpolation Method (NN Method)

In this method gray value of transformed grid point is estimated by the weighted average of gray values of nearest neighbours of transformed point in the reference image. Original intensities are preserved but resulting image has a blocky appearance. Nearest neighbour interpolation is unable to give sub voxel accuracy, as it is insensitive to translations up to one voxel.

Trilinear Interpolation

This may introduce new intensity values, which are originally not present in the reference image leading to unpredictable changes in marginal distribution functions. Use of Trilinear partial volume distribution overcomes this problem

This method is slower than NN method but resulting images are less blocky. Also it loses some high frequency information. A 2D version of this method is called the bilinear interpolation.

Partial Volume Interpolation (PV Method)

This method uses nearest neighbours of transformed point and weights but instead of finding a weighted average of neighbouring gray values and incrementing only one histogram entry, several histogram entries are increased by weight wi corresponding to the areas of cells so formed between the nearest neighbours of the pixel under consideration[Pluim,Maintz,Viergever(2000)]. No spurious gray levels are observed. Its advantage is that MI varies smoothly and interpolation artifacts are much reduced as compared to other methods.

Effect of interpolation on accuracy of Mutual Information based image registration showing smoothness in variation of MI has been illustrated in [Qi, Isao, Ukata 2006].

IMAGE DECOMPOSITION

To reduce the processing time in image registration, a coarse to fine strategy is employed in the search algorithm, which obtains finer resolution images from the coarse image. Image decomposition leads to a multi- resolution structure. At different resolutions details of the image are revealed. At coarse resolution normally larger structure information is revealed that gives the image context. So initially image is analyzed at coarse level and then we move high up in the pyramid towards finer resolution. Burt's Laplasian pyramid, wavelet based pyramid, cubic spline pyramid and pyramid obtained by down sampling [Hua-Mei, Varshney 2000] are some of the algorithms employed for image decomposition. In our experiments we have decomposed images using two methods Sub-sampling/filtering and wavelet based method and compared their performance on the basis of SNR.

B Decomposition By Sub-Sampling/Filtering

In this method image is sub sampled by Sub-sampling along rows and columns. Sub sampled image is then smoothed using filtering. By repeating sub sampling and smoothing operations we can achieve a Gaussian pyramid with subsequently lower resolution images. As the level increases, the resolution decreases by a factor of two. The size of image also reduces correspondingly. Again search space is reduced at each level to speed up the registration process. A sub-sampling rate of 2 and Weiner filter was used for the decomposition.

Wavelet Based Decomposition

Wavelet decomposition of a function $f(x) \in L^2(\Re)$ is defined as

$$(W_\psi f(x))(a,b) = \langle f(x), \psi_{ab}(x) \rangle$$
$$= 1/\sqrt{|a|} \int_\Re f(x), \psi((x-b)/a)\,dx$$

where $\psi_{ab}(x)$ defines the family of wavelet functions with $(a,b) \in \Re$ and a is the dilation parameter where $a \neq 0$ and b is the translation parameter.

Daubechies wavelet dB8 has been employed as it has the capability of keeping the energies in the low frequencies and low frequency sub bands are compact and smooth information of their original images and therefore are employed as searching spaces. As shown in Figure 1, to get a 2D multiscale pyramidal decomposition, a 2D image of size $2^n \times 2^m$ is convolved with H(Low pass) filter and G(High pass) filter in the horizontal direction. This is followed by down sampling along rows. The resulting images are then processed similarly along vertical direction ie. along the columns and down sampled. At the output four images are obtained, one at a coarser level, other three being horizontally, vertically and diagonally oriented images. The image at coarser level ie. LL can be applied as input to the next level for decomposition.

In our experiments we have decomposed the image into two levels in the registration hierarchy as blurredness of images increases at higher levels of decomposition. The idea behind decomposition is to reduce the size of search space. At each decomposition level image registration algorithm is applied.

PERFORMANCE COMPARISON

The performance of MI based Image Registration algorithm involving decomposition of images using Sub-sampling/filtering method and Discrete wavelet transform and is evaluated and performance is compared for a gray level Tea pot image of size 256x256. In Figure 2 (a) and 2(b) are the reference and test images where the test image 2(b) is a rotated and translated version of image 2(a). The reference and test images are both of size 256X256. In the two separate experiments, images were first decomposed into lower resolution images using Sub-sampling method and wavelet based decomposition method. The decomposed images were registered using maximization of Mutual Information. Maximum MI was determined from the maximum value of MI in the Mutual information matrix, which gave the MI values corresponding to given angle/angles of rotation and translation for each particular angle.

It was observed that Mutual Information (MI) and Signal to noise ratio (SNR) vary as a function of angle of rotation of the image about the registered position. Figure 3(a) shows that MI is maximum at the registered position and the MI values are higher in case of decomposition by Sub-sampling/filtering method in this first level of decomposition.

Variation of MI in Tea pot images so registered as a function of angle of rotation for the second level of decomposition is shown in Figure.3 (b) and it is seen that MI is higher with wavelet decomposition method rather than Sub-sampling/filtering method. Also MI is seen to vary smoothly with this method compared to that with Sub-sampling method.

Variation of SNR in the case of Teapot images so registered using the two decomposition methods for the first level of decomposition method is shown in Figure.3(c). Here SNR is also seen to be high for the Sub-sampling/filtering method compared to wavelet decomposition method, as was the variation of MI under similar conditions.

Variation of SNR as a function of angle of angle of rotation using wavelet decomposition is as shown in Figure3 (d). Here SNR is seen to be much more than that with sub sampling method of decomposition.

SNR in the two levels of decompositions for the two methods is calculated as

$$SNR=10\log_{10}[(signal)^2/(noise)^2]$$

where the signal when decomposition was applied is the decomposed reference image and noise is the difference of decomposed reference image at a given level and the registered (matched) image at that level. The difference between the original reference image and test image is obtained as the difference image, which is as shown in Figure. 4(a)

Observations regarding SNR can also be verified from the reduction in noise from difference images so obtained at the two decomposition levels respectively as shown in Figure.4 (b)-(e). Figure.4 (b) shows the difference image after registration at first decomposition level using Sub-sampling/filtering.

The differences in the images are slightly reduced at second level of decomposition on registering the Teapot images using Sub-sampling/filtering method. This is as shown in Figure.4(c)

When wavelet decomposition was used for hierarchical decomposition while registering images, the difference image so obtained at level 1 is as shown in Figure.4 (d).

Again differences at level 1 using wavelet decomposition are more compared to that of differences at level 1 in case of Sub-Sampling/filtering. However, moving to second level of decomposition using Wavelet decomposition, these differences are greatly reduced and hence the noise. This results in an improvement in the SNR as level of decomposition increases and by use of wavelet decomposition. This is clear from the difference image as shown in Figure.4 (e)

SNR was determined for various methods such as between the original images A and B without filtering and decomposition being applied, then between the same two images where they were initially filtered but image decomposition was not applied, in the third method SNR was found at the registered position when Sub-sampling/filtering method was used and lastly between the said images at the registered position for the case when wavelet decomposition was applied. It was observed that among all these methods, SNR was the highes for images registered using wavelet decomposition as shown in table. 1.

CONCLUSION

Use of Discrete wavelet transform fo hierarchical image decomposition can no only help in improving the computationa efficiency but also give considerabl increase in SNR in the images so registerec This can be useful in registration of noisy images.

In this paper image registration was carried out on "Tea pot images and it was observed that for a fix step size which is th size by which pixel translations are estimated and varyin angle of rotation within [-5 to +5] radians about the registere position, MI value increases for higher levels of th registration pyramid in both the methods. MI is higher in cas of registration of wavelet decomposed images rather tha image decomposition using Sub-sampling /filtering. Howeve range over which Mutual information varies is less. Variatio of MI is smooth in case of wavelet-decomposed images. Signa to noise ratio at second level of decomposition was found to b high as the differences at this level were much reduced.

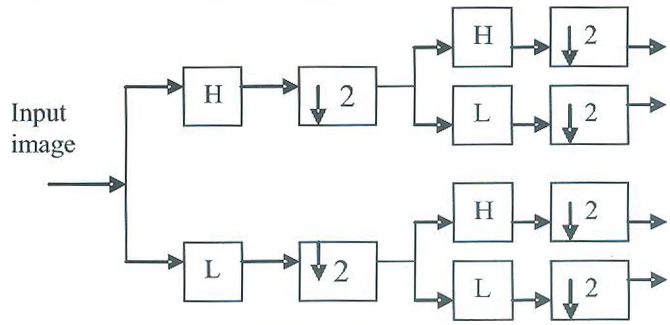

Figure 1.
Single level wavelet 2D decomposition

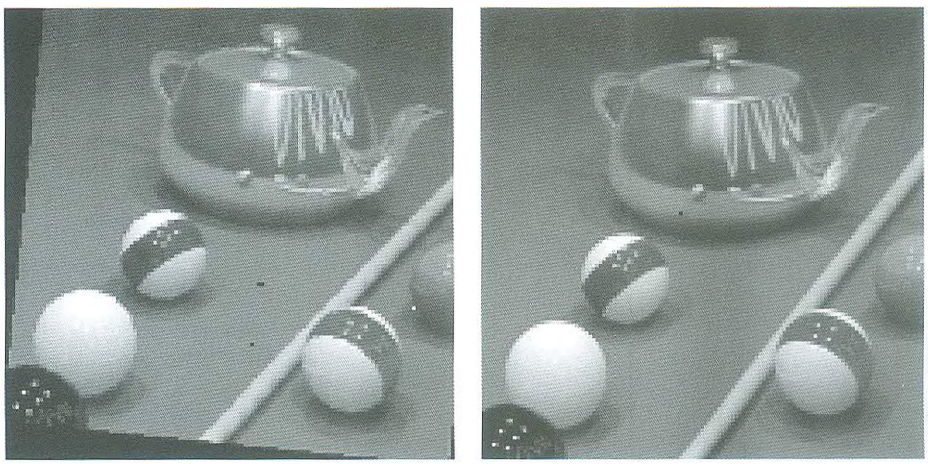

Figure 2 . (a) Reference image (b) Test image

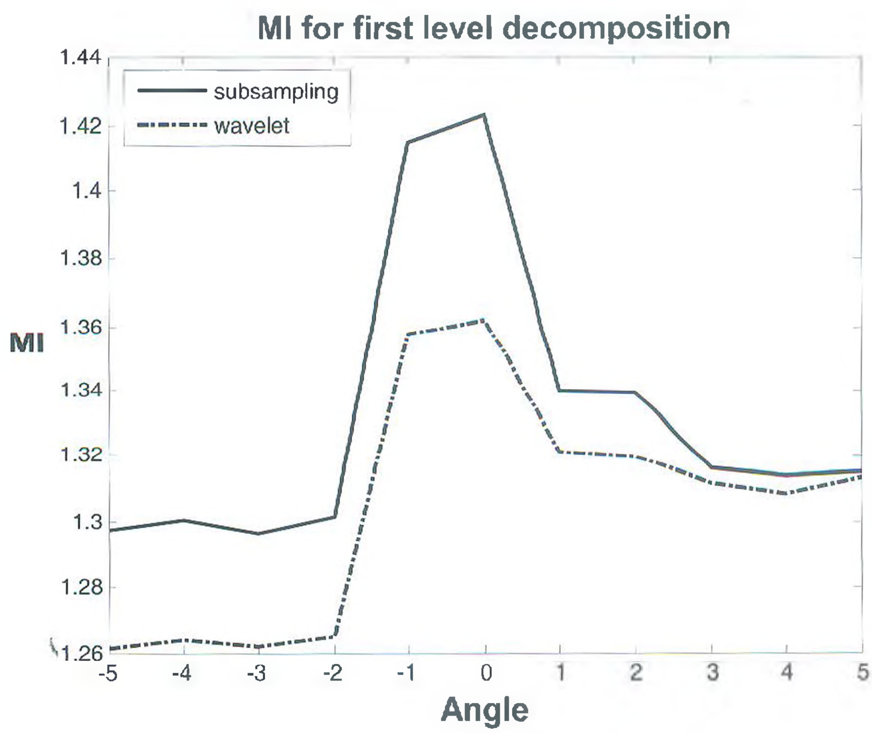

Figure 3(a)
Variation of MI as function of angle of rotation for first level of decomposition.

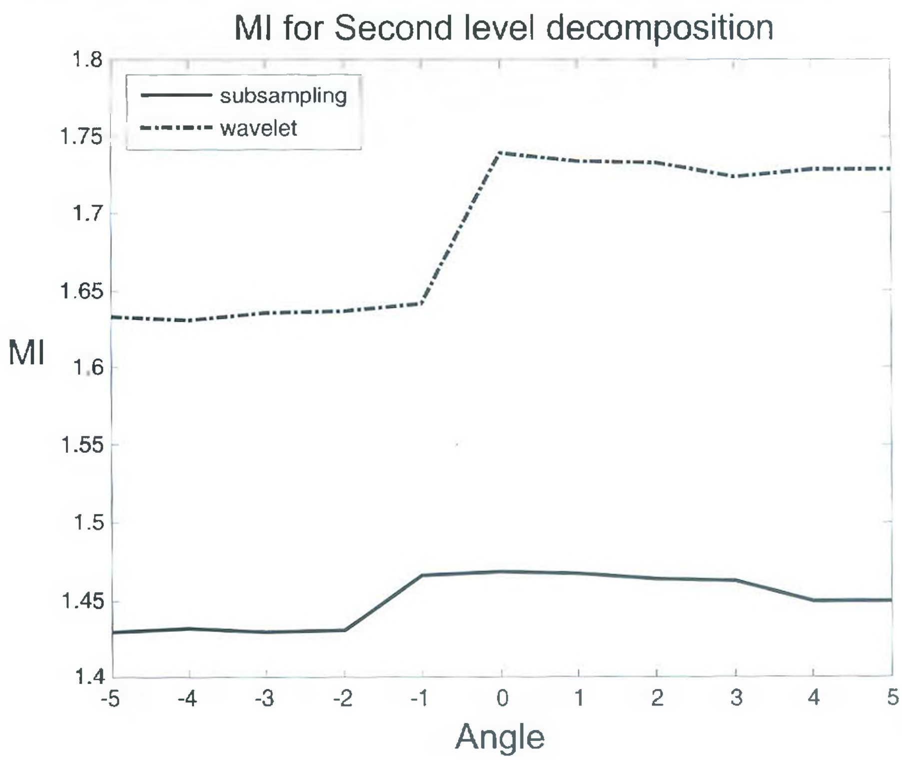

Figure 3(b)
Variation of MI as function of angle of rotation for second level of decomposition

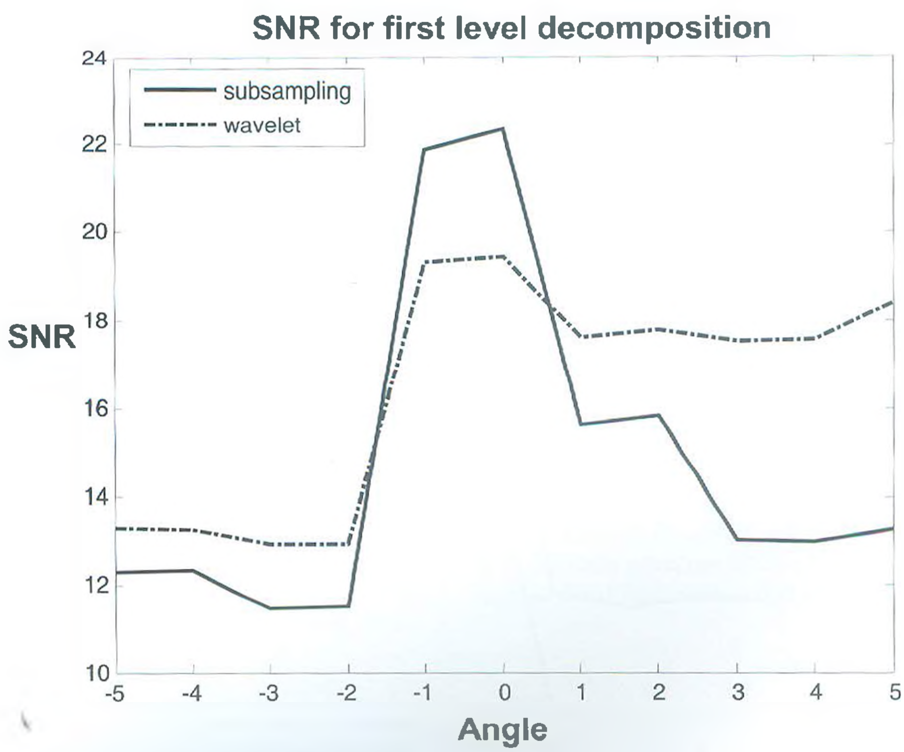

Figure 3(c)
Variation of SNR as a function of angle of rotation for first level of decomposition

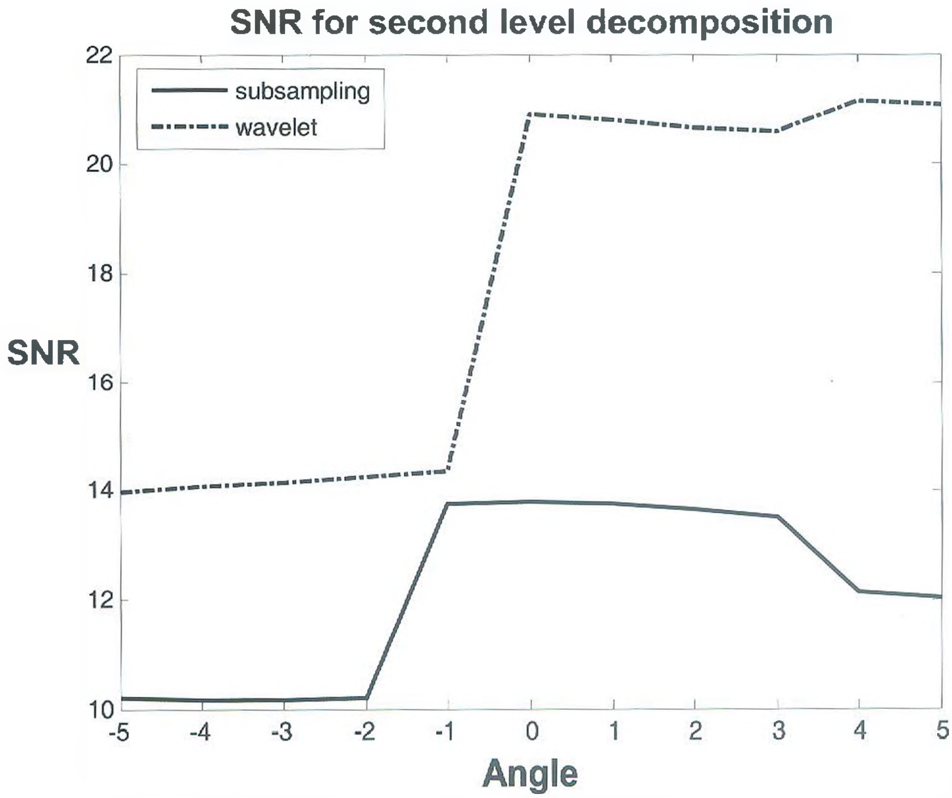

Figure 3(d)
variation of SNR as a function of angle of rotation for second level of Decomposition

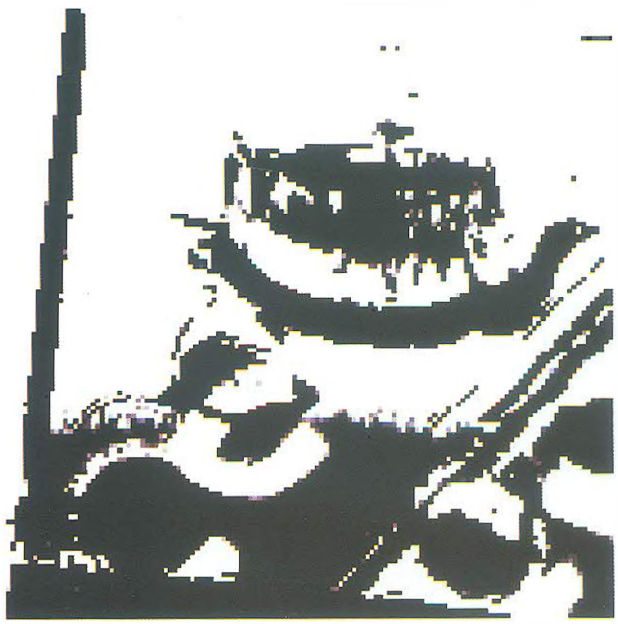

Figure 4.(a)
Difference image without registration

Figure 4.(b)
Difference image at level 1 after registration using Sub-Sampling/filtering

Figure 4. (c)
Difference image at level 2 after registration using Sub-sampling/filtering.

Figure 4. (d)
Difference image at level 1 after registration using Wavelet decomposition.

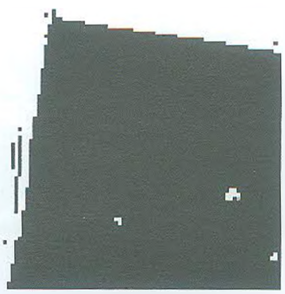

Figure 4.(e)
Difference image at level 2 after registration using Wavelet decomposition.

Methods	SNR
Original images without filtering and decomposition	10.284
Filtered images without decomposition	10.388
Sub-sampled and filtered Images	13.78
Wavelet Decomposed Images	20.778

Table 1.

Comparison of SNR values for "Tea pot" images using different methods.

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