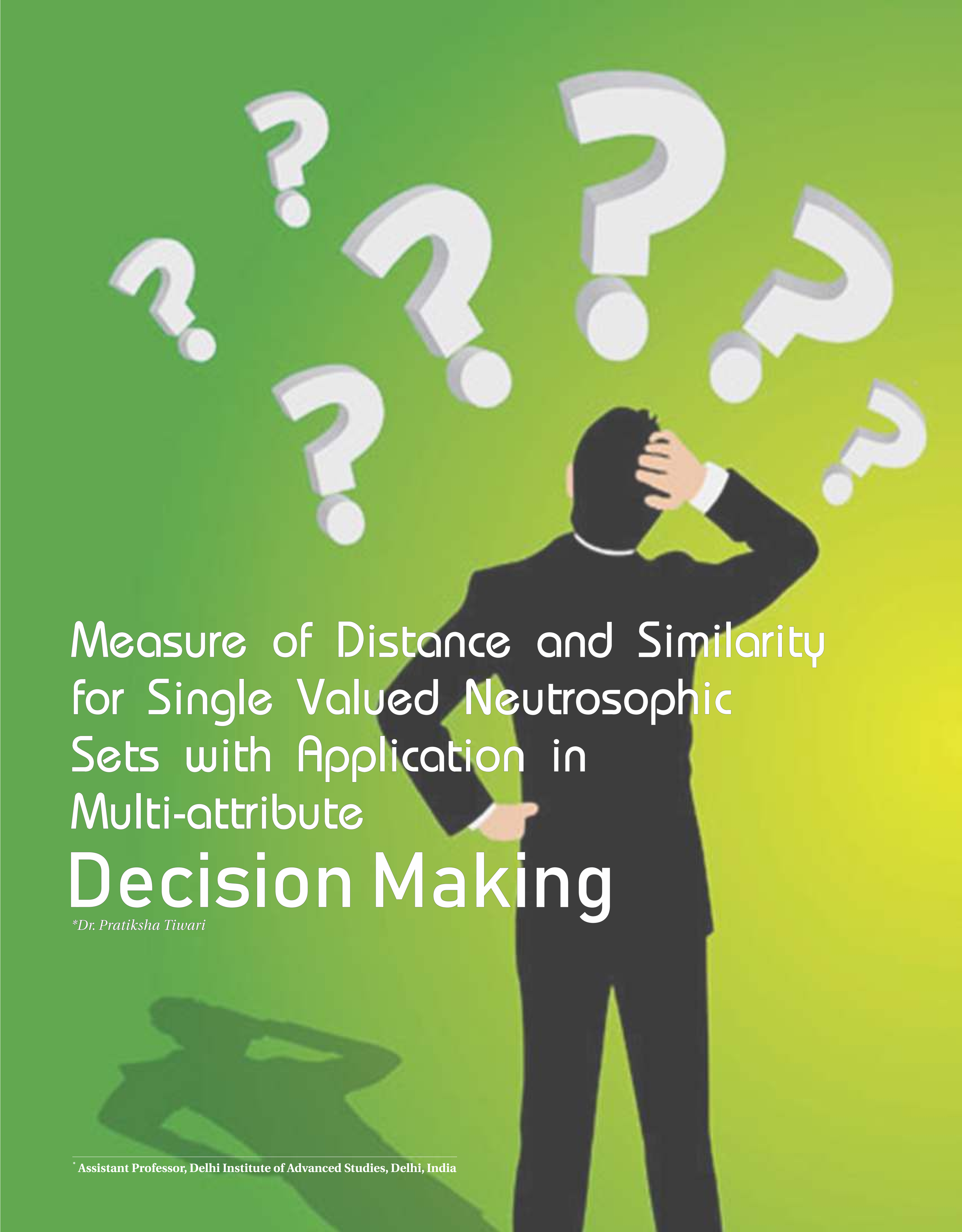

Measure of Distance and Similarity for Single Valued Neutrosophic Sets with Application in Multi-attribute Decision Making

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ABSTRACT

A single valued neutrosophic sets (SvNSs) is a particular case of neutrosophic set, which can handle scientific and engineering applications in real world. Distance, similarity, entropy and cross entropy measures of SvNSs are major tool is to solve various real world problems In the paper, an average distance measure based on Hamming and Hausdorff distance is defined for SvNSs. Similarity measure is proposed on the basis of defined averaged distance using the relationship between distance and similarity measures. Further, one more similarity measure is defined and its validity is proved. Then a multi-attribute decision making is also established under single valued neutrosophic environment, in which attribute values are assigned to each course of action by the decision makers/experts and attribute weights are known, whereas the optimal point is defined using the decision matrix using the proposed method. Also, an algorithm is presented to determine the ranking of available course of actions on the basis of values of measure of similarity amongst optimal point and the available course of actions. Lastly, an example is used to demonstrate the application of proposed similarity measures in multi-attribute decision making.

INTRODUCTION

Neutrosophic set developed by Smarand ache (1999) is a new evolving instrument for uncertain data processing. It has the potential of being a framework for analysis of uncertain data sets which include big data sets. Neutrosophic sets are generalization of interval valued intuitionistic fuzzy sets. Because of the fuzziness and uncertainty of many practical problems in the real world, it is applicable to a wide range of practical problems. Zadeh (1965) developed fuzzy set theory which is a huge success in various areas involving uncertainty. In fuzzy set theory, non-membership value of an element is defined normally as complement of its membership value from one, but practically it is not so. This situation is dealt by higher order fuzzy sets proposed by Atanassov (1986) and is termed as intuitionistic fuzzy sets (IFSs). It is found to be highly useful in dealing vagueness and hesitancy originated from inadequate information. It characterizes three characteristic functions for membership and non-membership and hesitancy respectively for an element of the universe of discourse where sum of all the three functions is 1. Therefore, IFSs are much more flexible and practical than fuzzy sets in dealing with vagueness and uncertainty problems. However, fuzzy sets, IFSs, cannot handle indeterminate information and inconsistent information that exist in the real world and we need further structures such as neutrosophic sets which is a powerful generalization of classic sets, fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets, interval valued intuitionistic fuzzy sets, paraconsistent sets, dialetheist set, paradoxist sets, and tautological sets. Like intuitionistic fuzzy sets,neutrosophic sets are also characterized by three functions: truth-membership, indeterminacy-membership, and falsity-membership, but they are represented independently. The neutrosophic sets generalize the above-mentioned sets from a philosophical point of view and its functions are real standard or non-standard subsets of]–0, 1+[, and there is no restriction on the sum of three. Thus, it will be difficult to apply these in real scientific and engineering areas. Thus, Wang et al. (2010) derived single-valued neutrosophic set (SvNS), which is a particular case of neutrosophic set. It can describe and handle indeterminate information and inconsistent information. For example, when we ask a customer for his/ her opinion about a statement, he/she may say that the possibility of agreeing to a statement is 0.6 and disagreeing is 0.4 and not sure is 0.2. For a SvNS notation it can be represented as (0.6, 0.4, 0.2) whereas it cannot be dealt by intuitionistic fuzzy sets as sum of membership, non-membership and hesitation value is not 1. Therefore, the notion of a neutrosophic set is more general and overcomes the aforementioned issues.

Various researcher studied different measures such as entropy, cross entropy, distance and similarity for fuzzy and intuitionistic fuzzy sets that will help in decision making. The concept of similarity is primarily significant in practically every field. Many methods have been proposed for measuring the degree of similarity between fuzzy and intuitionistic fuzzy sets. But these methods are not suitable for dealing with the similarity measures of neutrosophic set. Few researchers have dealt with the similarity measures for neutrosophic set. This paper deals with the similarity measures for single valued neutrosophic sets and demonstrates its use in multi-attribute decision making, using an illustrative example.

BASIC CONCEPTS AND RELATED WORK

Neutrosophic sets emerged as a tool to deal uncertain data. It has the potential to become a general framework for uncertainty analysis in data. The neutrosophic set is a part of neutrosophy and generalizes FS, IvFS, IFS, and IvIFS from a philosophical point of view. Smarandache (1999) derived neutrosophic set which is defined as follows:

Definition 1: Let X be a universe of discourse, a neutrosophic set A in X is characterized by a quadruple $\langle x, T_A(x), I_A(x), F_A(x) \rangle$ i.e. (x, truth-membership function, indeterminacy-membership function, falsity-membership function), where x X. The functions $T_A(x)$: X]–0, 1 + [, $I_A(x)$,: X]–0, 1 +[, and $F_A(x)$: X]–0, 1 +[. Thus there is no restriction on the sum so $-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+$.

Wang et al. (2011) proposed a subclass of neutrosophic sets termed as single valued neutrosophic sets which are easier to apply to real scientific and engineering problems and is defined as

Definition 1.1: Let X universe of discourse, a single valued neutrosophic set (SvNS) A in X is characterized by quadruple $\langle x, T_A(x), I_A(x), F_A(x)\rangle$ i.e. (x, truth-membership function, indeterminacy-membership function, falsity-membership function), where $x \in X$, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2 (Set operations on single valued neutrosophic sets): Let A and B be two SvNSs defined by quadruple x, $T_A(x)$, I_A (x), $F_A(x)$ and x, $TB(x)$, $IB(x)$, $F_B(x)$ respectively, where $x \in X$ then set operations are defined as follows:

1) $A \cup B = \{\langle x, T_A(x) \vee T_B(x), I_A(x) \vee I_B(x), F_A(x) \wedge F_B(x)\rangle / x \in X\}$

2) $A \cap B = \{+x, T_A(x) \wedge T_B(x), I_A(x) \wedge I_B(x), F_A(x) \vee F_B(x)\rangle / x \in X\}$

3) $\bar{A} - \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle / x \in X\}$

4) $A \subseteq B$ iff $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$ and $F_A(x) \geq F_B(x)\ \forall\, x \in X$;

5) $A = B$ iff $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$ and $F_A(x) = F_B(x)\ \forall\, x \in X$.

Definition 3 (Distance between two single valued neutrosophic sets): For any two SvNSs A and B, a real valued function D: SvNSs (X) × SvNSs () [0, 1] is termed as a distance measure of SvNSs on X, if it satisfies the below mentioned axioms:

1) Distance between any two SvNSs A and B is zero if A = B.

2) Distance measure is symmetrical w.r.t to any two SvNSs A and B.

3) For any three SvNSs A, B and C such that A B C, we have $D(A, C)$ $D(A, B)$ and $D(A, C)$ $D(B, C)$.

Distance between FSs was presented by (Kacprzyk, 1997). Its extension was proposed by Atanassov in 1999 as two dimensional distances whereas third parameter hesitancy degree in distance was introduced by Szmidt and Kacprzyk (2000) for intuitionistic fuzzy sets. Yang & Chiclana (2012) proved three dimensional distance consistency over two dimensional distances. Grzegorzewski (2004) and Park et al. (2007) gave distance measure for IvFSs and IvIFSs respectively. Broumi and Smarandache (2013) presented weighted Hausdorff distance measure based on Hausdorff distance.

$$D_i(A,B)=\frac{1}{n}\sum_{i=1}^{n} w_i \left| |T_A(x_i)-T_B(x_i)| \vee |I_A(x_i)-I_B(x_i)| \vee |F_A(x_i)-F_B(x_i)| \right| \quad \dots(1)$$

Ye (2014) presented weighted distance measures for SvNSs based on Hamming and Euclidean Distances given below:

Weighted Hamming Distance

$$D_1(A,B)=\frac{1}{3}\sum_{i=1}^{n} w_i \left| |T_A(x_i)-T_B(x_i)| + |I_A(x_i)-I_B(x_i)| + |F_A(x_i)-F_B(x_i)| \right| \quad \dots(2)$$

where w_i is weight corresponding to x_i such that $\sum_{i=1}^{n} w_i = 1$. And weighted Euclidean Distance $D_2(A, B)$

$$=\left\{\frac{1}{3}\sum_{i=1}^{n} w_i \left[(T_A(x_i)-T_B(x_i))^2+(I_A(x_i)-I_B(x_i))^2+(F_A(x_i)-F_B(x_i))^2\right]\right\}^{1/2} \quad \dots(3)$$

where w_i is weight corresponding to x_i such that $\sum_{i=1}^{n} w_i = 1$.

Definition 4 (Similarity between SvNSs) : Let A and B be any two SvNSs, a real valued function S: IvISs () × IvIFSs () [0, 1] is defined as a measure of similarity for IvIFSs on X, if it satisfies following axioms:

1) Measure of similarity between any two IvIFSs is 1 iff A = B.

2) Measure of similarity is symmetric w.r.t. any two IvIFSs.

3) For any three IvIFSs A, B and C such that A B C. We have $S(A, C)$ $S(A, B)$ and $S(A, C)$ $S(B, C)$.

From axiomatic definition of distance and similarity measures it is clear that $S(A, B) = 1 - D(A, B)$ where A and B are SvNSs, D and S are distance and similarity measure for SvNSs respectively.

Broumi and Smarandache (2013) presented several similarity measures for SvNSs. Ye (2014) presented similarity measures based on equation (2) and (3) as

$$S_1(A,B)=1-\left\{\frac{1}{3}\sum_{i=1}^{n} w_i \begin{bmatrix} |T_A(x_i)-T_B(x_i)| \\ +|I_A(x_i)-I_B(x_i)| \\ +|F_A(x_i)-F_B(x_i)| \end{bmatrix}\right\}$$

and

$$S_2(A,B)=1-\left\{\frac{1}{3}\sum_{i=1}^{n} w_i \begin{bmatrix} (T_A(x_i)-T_B(x_i))^p \\ +(I_A(x_i)-I_B(x_i))^p \\ +(F_A(x_i)-F_B(x_i))^p \end{bmatrix}\right\}^{1/p}.$$

Ye also presented one more similarity measure as

$$S_3(A,B)=\frac{1-\left\{\frac{1}{3}\sum_{i=1}^{n} w_i\left[(T_A(x_i)-T_B(x_i))^p+(I_A(x_i)-I_B(x_i))^p-(F_A(x_i)-F_B(x_i))^p\right]\right\}^{1/p}}{1+\left\{\frac{1}{3}\sum_{i=1}^{n} w_i\left[(T_A(x_i)-T_B(x_i))^p+(I_A(x_i)-I_B(x_i))^p-(F_A(x_i)-F_B(x_i))^p\right]\right\}^{1/p}}$$

Ye (2015) proposed improved measures of cosine similarity measures for simplified neutrosophic sets including single valued cosine similarity and interval valued neutrosophic cosine similarity. He also introduced corresponding weighted similarity measures and applied it to medical diagnoses. Aydoğdu (2015) introduced two measures of similarity for SvNSs and developed single entropy measure for the same and applied it to neutrosophic multi-criteria decision making. Ye &Smarandache (2016) introduced refined single-valued neutrosophic sets, developed similarity measure of the same and applied it to multi criteria decision making.

Szmidt and Kacprzyk (2009) constructed Hausdorff distance between Intuitionistic Fuzzy Sets based on the Hamming metric and particularly give importance to the consistency of the metric used and the essence of the Hausdorff distances. Next section deals with new weighted distance and similarity measures based on averaged distance measure based on Hamming and Hausdorff distance defined as

$$D_4(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\left\{\begin{matrix}\frac{|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|}{3} \\ +|T_A(x_i)-T_B(x_i)| \vee |I_A(x_i)-I_B(x_i)| \vee |F_A(x_i)-F_B(x_i)|\end{matrix}\right\} \quad \dots(4)$$

Distance, similarity measures and optimal point is proposed in the following sections under single valued neutrosophic environment.

Distance and Similarity Measures for SvNSs:

This section defines weighted distance and similarity measures based on equation (4) as follows:

Definition 5 Distance Measure: Consider two SvNSs A and B, represented by quadruple $\langle x, T_A(x), I_A(x), F_A(x)\rangle$ and $\langle x, T_B(x), I_B(x), F_B(x)\rangle$ respectively in a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$. Weighted distance measure $D_W(A, B)$ based on averaged distance measure is defined as follows:

$$D_W(A,B)=\frac{1}{2n}\sum_{i=1}^{n} w_i\left\{\begin{matrix}\frac{|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|}{3} \\ +|T_A(x_i)-T_B(x_i)| \vee |I_A(x_i)-I_B(x_i)| \vee |F_A(x_i)-F_B(x_i)|\end{matrix}\right\}, \quad \dots(5)$$

where wi is weight corresponding to xi such that $\sum_{i=1}^{n} w_i = 1$.

Theorem 1: Dw(A, B) is a valid measure of distance between two SvNSs A and B.

Proof: Consider two SvNSs A and B represented by quadruple $\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle$ and $\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle$ respectively in a universe of discourse = $\{x_1, x_2, \dots, x_n\}$. It is easy to see that $D_W(A,B)$ [0, 1] satisfies axioms (1) and (2) defined in Definition 3. In order to prove axiom (3), consider three SvNSs A, B and C such that $A \subseteq B \subseteq C$.

Then

$$T_A(x_i) \le T_B(x_i) \le T_C(x_i),\ I_A(x_i) \ge I_B(x_i) \ge I_C(x_i) \text{ and } F_A(x_i) \ge F_B(x_i)) \ge F_C(x_i). \text{ Thus}$$
$$|T_A(x_i) - T_B(x_i)| \le |T_A(x_i) - T_C(x_i)| \text{ and } |T_B(x_i) - T_C(x_i)| \le |T_A(x_i) - T_C(x_i)|,$$
$$|I_A(x_i) - I_B(x_i)| \le |I_A(x_i) - I_C(x_i)| \text{ and } |I_B(x_i) - I_C(x_i)| \le |I_A(x_i) - I_C(x_i)|,$$
$$|F_A(x_i) - F_B(x_i)| \le |F_A(x_i) - F_C(x_i)| \text{ and } |F_B(x_i) - F_C(x_i)| \le |F_A(x_i) - F_C(x_i)|.$$

$$\Rightarrow \frac{\begin{bmatrix} |T_A(x_i) - T_B(x_i)| \\ +|I_A(x_i) - I_B(x_i)| \\ +|F_A(x_i) - F_B(x_i)| \end{bmatrix}}{3} \le \frac{\begin{bmatrix} |T_A(x_i) - T_C(x_i)| \\ +|I_A(x_i) - I_C(x_i)| \\ +|F_A(x_i) - F_C(x_i)| \end{bmatrix}}{3}$$

And

$$\frac{\begin{bmatrix} |T_B(x_i) - T_C(x_i)| \\ +|I_B(x_i) - I_C(x_i)| \\ +|F_B(x_i) - F_C(x_i)| \end{bmatrix}}{3} \le \frac{\begin{bmatrix} |T_A(x_i) - T_C(x_i)| \\ +|I_A(x_i) - I_C(x_i)| \\ +|F_A(x_i) - F_C(x_i)| \end{bmatrix}}{3}$$

Also, $$\begin{bmatrix} |T_A(x_i) - T_B(x_i)| \\ \vee\ |I_A(x_i) - I_B(x_i)| \\ \vee\ |F_A(x_i) - F_B(x_i)| \end{bmatrix} \le \begin{bmatrix} |T_A(x_i) - T_C(x_i)| \\ \vee\ |I_A(x_i) - I_C(x_i)| \\ \vee\ |F_A(x_i) - F_C(x_i)| \end{bmatrix}$$

$$\begin{bmatrix} |T_B(x_i) - T_C(x_i)| \\ \vee\ |I_B(x_i) - I_C(x_i)| \\ \vee\ |F_B(x_i) - F_C(x_i)| \end{bmatrix} \le \begin{bmatrix} |T_A(x_i) - T_C(x_i)| \\ \vee\ |I_A(x_i) - I_C(x_i)| \\ \vee\ |F_A(x_i) - F_C(x_i)| \end{bmatrix}$$

Thus $D_w(A, B)$ and $D_w(B, C)$ $D_w(A, C)$. Hence $D_w(A, B)$ is a valid measure of distance.

According to the relationship between the distance and similarity measures, similarity measure corresponding to $D_w(A, B)$ is derived as follows

$$S_w(A, B) = 1 - \frac{1}{2n}\sum_{i=1}^{n} w_i \left\{ \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3} + |T_A(x_i) - T_B(x_i)| \vee |I_A(x_i) - I_B(x_i)| \vee |F_A(x_i) - F_B(x_i)| \right\}, \quad \ldots(6)$$

where w_i is weight corresponding to x_i such that $\sum_{i=1}^{n} w_i = 1$.

Obviously, we can easily prove that $S_w(A, B)$ satisfies the axioms mentioned in Definition 4. Furthermore, we can also propose another measure of similarity for SvNSs.

$$S(A, B) = \frac{1}{n}\sum_{i=1}^{n} \frac{T_{A\cap B}(x_i) + I_{A\cap B}(x_i) + F_{A\cap B}(x_i)}{T_{A\cup B}(x_i) + I_{A\cup B}(x_i) + F_{A\cup B}(x_i)} \quad \ldots(7)$$

Theorem 2: $S(A, B)$ is a valid measure of similarity between two SvNSs A and B.

Proof: Consider two SvNSs A and B represented by quadruple $\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle$ and $\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle$ respectively in a universe of discourse = $\{x_1, x_2, \ldots, x_n\}$. It is easy to see that $S(A, B) \in [0, 1]$ satisfies axioms (1) and (2) defined in Definition 4. In order to prove axiom (3), assume that A, B and C are three SvNSs such that $A \subseteq B \subseteq C$. Then $T_A(x_i) \le T_B(x_i) \le T_C(x_i)$, $I_A(x_i) > I_B(x_i) \ge I_C(x_i)$ and $F_A(x_i) \ge F_B(x_i)) \ge F_C(x_i)$. Thus

$$T_{A\cap B}(x_i) = T_A(x_i) \wedge T_B(x_i) = T_A(x_i) = T_A(x_i) \wedge T_C(x_i) = T_{A\cap C}(x_i) \quad \ldots(8)$$

$$I_{A\cap B}(x_i) = T_A(x_i) \wedge I_B(x_i) = I_B(x_i) \ge I_C(x_i) = I_A(x_i) \wedge I_C(x_i) = I_{A\cap C}(x_i) \quad \ldots(9)$$

And

$$F_{A\cap B}(x_i) = F_A(x_i) \vee F_B(x_i) = F_A(x_i) = F_A(x_i) \vee F_C(x_i) = F_{A\cap C}(x_i) \quad \ldots(10)$$

Adding equations (8), (9) and (10)

$$T_{A\cap B}(x_i) + I_{A\cap B}(x_i) + F_{A\cap B}(x_i) \ge T_{A\cap C}(x_i) + I_{A\cap C}(x_i) + F_{A\cap C}(x_i) \quad \ldots(11)$$

Also,

$$T_{A\cup B}(x_i) = T_A(x_i) \vee T_B(x_i) = T_B(x_i) \le T_C(x_i) = T_A(x_i) \vee T_C(x_i) = T_{A\cup C}(x_i) \quad \ldots(12)$$

$$I_{A\cup B}(x_i) = I_A(x_i) \vee I_B(x_i) = I_A(x_i) = I_A(x_i) \vee I_C(x_i) = I_{A\cup C}(x_i) \quad \ldots(13)$$

And

$$F_{A\cup B}(x_i) = F_A(x_i) \wedge F_B(x_i) = F_B(x_i) \ge F_C(x_i) = F_A(x_i) \wedge F_C(x_i) = F_{A\cup C}(x_i) \quad \ldots(14)$$

Adding equations (12), (13) and (14)

$$T_{A\cup B}(x_i) + I_{A\cup B}(x_i) + F_{A\cup B}(x_i) \le T_{A\cup C}(x_i) + I_{A\cup C}(x_i) + F_{A\cup C}(x_i) \quad \ldots(15)$$

$$\Rightarrow \frac{1}{T_{A\cup B}(x_i) - I_{A\cup B}(x_i) - F_{A\cup B}(x_i)} \ge \frac{1}{T_{A\cup C}(x_i) + I_{A\cup C}(x_i) + F_{A\cup C}(x_i)} \quad \ldots(16)$$

From inequalities (11) and (16), we get

$$\frac{T_{A\cap B}(x_i) + I_{A\cap B}(x_i) - F_{A\cap B}(x_i)}{T_{A\cup B}(x_i) + I_{A\cup B}(x_i) - F_{A\cup B}(x_i)} \ge \frac{T_{A\cap C}(x_i) + I_{A\cap C}(x_i) - F_{A\cap C}(x_i)}{T_{A\cup C}(x_i) + I_{A\cup C}(x_i) - F_{A\cup C}(x_i)}$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} \frac{T_{A\cap B}(x_i) + I_{A\cap B}(x_i) + F_{A\cap B}(x_i)}{T_{A\cup B}(x_i) + I_{A\cup B}(x_i) + F_{A\cup B}(x_i)} \ge \frac{1}{n}\sum_{i=1}^{n} \frac{T_{A\cap C}(x_i) + I_{A\cap C}(x_i) + F_{A\cap C}(x_i)}{T_{A\cup C}(x_i) + I_{A\cup C}(x_i) + F_{A\cup C}(x_i)}$$
$$\Rightarrow S(A, C) \le S(A, B)$$

Similarly, $S(A, C) \le S(B, C)$. Thus $S(A, B)$ is a valid measure of similarity between two SvNSs.

Corollary: Weighted similarity measure corresponding to equation (7) can be defined as

$$S^w(A, B) = \frac{1}{n}\sum_{i=1}^{n} \frac{T_{A\cap B}(x_i) + I_{A\cap B}(x_i) + F_{A\cap B}(x_i)}{T_{A\cup B}(x_i) + I_{A\cup B}(x_i) + F_{A\cup B}(x_i)} \quad \ldots(17)$$

Next, section defines an optimal point and presents an algorithm that helps to solve multi-attribute decision making problems.

Application of SVNSs Similarity Measures to Multi-attribute Decision Making

This section proposes definition of optimal point and uses proposed similarity measures to draw inferences in multi attribute decision making under single valued neutrosophic environments.

Multi-attribute decision making in an organization involves various attributes along with various decision takers. Recently, it has become more complex. To take any decision, a manager needs to have relevant information and decent analytical skills. In some practical situations the course of action involves incomplete and indeterminate information, which is stated in terms of SvNSs. Similarity measures can be used as a tool to identify best course of action by determining the similarity between each course of action and ideal decision criteria/ideal point. Ideal point does not exist in reality. In order to identify the substitute of ideal point we have defined optimal point as follows:

$$A_* = \{\langle x_i, \max[T(x_i)], \max[I(x_i)], \min[F(x_i)]\rangle, \forall I\} \qquad \ldots(18$$

The best course of action can be identified on the basis of similarity values between optimal point and available course of actions. Larger the value of similarity more closer it is to the optimal point.

Let us consider a multi-attribute decision making problem involving a set of options P = {P_1, P_2, … … … ., P_m} to be considered on the basis of attributes C = {C_1, C_2, … … … ., C_n}. Assume that the weight of an attribute C_j (j = 1, 2, . . . , n), entered by the decision-maker, is $w_j \in [0, 1]$, j = 1, … , n and $\sum_{i=1}^{n} w_i = 1$. Corresponding to each option P_i, I = 1, … , m and attribute C_j, j = 1,2, … , n, the values of three function TA(xi), $I_A(x_i)$, and $F_A(x_i)$ denoted by single valued neutrosophic value $d_{ij}(x_{ij})$, which is derived from evaluation of each course of action based on each attribute. To identify best course of action, similarity between each course of action and identified optimal course of action is calculated. Higher the value of similarity measure closer it is with the optimal value. Course of action with highest value is identified as best course of action.

Multi-attribute decision process can be summarized as follows:

Step 1: Identify weight corresponding to each attribute.

Step 2: Formulate decision matrix corresponding to each attribute provided by the decision maker.

Step 3: Identify optimal point using equation (18) using decision matrix obtained in step 2.

Step 4: Calculate the similarity value $S_w(P_j, A_*)$ or $S(P_j, A_*)$ or $S^w(P_j, A_*)$ by using equation (6) or (7) or (17).

Step 5: Rank and identify the best alternatives on the basis weighted similarity measure value.

Next subsection explains the procedure of application of similarity measure in multi-attribute decision making using a numerical example.

LLUSTRATIVE EXAMPLE

Here multi-attribute decision making problem is adopted from Ye (2014) to demonstrate the procedure of multi-attribute decision making. "Consider four suppliers P = {P_1, P_2, P_3, P_4} which are concerned with a manufacturing company, that wants to select the best global supplier according to the core competencies of suppliers. The core competencies of suppliers are evaluated on the basis of four attributes: (i) C_1 is the level of technology innovation; (ii) C_2 is the control ability of the flow; (iii) C_3 is the ability of management; (iv) C_4 is the level of service. Then, the weight vector for the four criteria is w = (0.3, 0.25, 0.25, 0.2)^T. The decision matrix of the suppliers is made according to the four evaluating criteria. Therefore, the single valued neutrosophic decision matrix of the suppliers is as follows:

$$D = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{bmatrix} \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.7,0.1,0.2\rangle & \langle 0.3,0.2,0.1\rangle \\ \langle 0.4,0.2,0.3\rangle & \langle 0.3,0.2,0.4\rangle & \langle 0.9,0.0,0.1\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.4,0.3,0.1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.0,0.4\rangle & \langle 0.6,0.2,0.2\rangle \\ \langle 0.6,0.1,0.2\rangle & \langle 0.2,0.2,0.5\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.7,0.2,0.1\rangle \end{bmatrix}$$

To identify the most desirable supplier, we calculate the similarity of each supplier with optimal identified values.

Optimal solution is identified using equation (18) as follows

$A_* = \{\langle 0.6, 0.3, 0.1\rangle, \langle 0.5, 0.2, 0.3\rangle, \langle 0.9, 0.3, 0.1\rangle, \langle 0.7, 0.3, 0.1\rangle\}$

$S^w(A_*, A_1) = 0.821399$, $S^w(A_*, A_2) = 0.869126$, $S^w(A_*, A_3) = 0.819895$, $S^w(A_*, A_4) = 0.849895$, optimal solution is identified as the alternative with maximum value of similarity measure. So, the manufacturing company should order from supplier A_2 with preference order A_4, A_1, A_3. According to distance similarity measure, $S_w(A_*, A_1) = 0.955417$, $S_w(A_*, A_2) = 0.956875$, $S_w(A_*, A_3) = 0.957917$, $S_w(A_*, A_4) = 0.948958$, optimal solution is identified as the alternative with maximum value of similarity measure. So, the manufacturing company should order from supplier A_3 with preference order A_2, A_1, A_4.

It can be easily observed that the cross entropy measures proposed by Ye (2014) and Ye (2016) provide ranking A_1, A_3, A_2, A_4 and A_3, A_1, A_2, A_4 respectively to the suppliers. But Ye (2014) and Ye (2016) had taken optimal solution as ⟨1,0,0⟩ which is quite unrealistic. So, the ranking provided by the proposed similarity measure is more reasonable as the optimal solution considered in this paper is more realistic.

- $S_{BS}(A,B) = 1 - D(A,B)$, where $D(A,B)$ is Hausdorff distance between A and B as proposed by Broumi & Smarandache (2013).
- $S_{BS1}(A,B) = \dfrac{\sum_{i=1}^{n}(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{\max\left(\sum_{i=1}^{n}(T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2, \sum_{i=1}^{n}(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2\right)}$ presented by Broumi & Smarandache (2013).
- $S_J(A,B) = \frac{1}{n}\sum_{i=1}^{n} \dfrac{T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)}{\begin{pmatrix}((T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2) \\ +(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2 \\ -(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))\end{pmatrix}}$, $S_D(A,B) = \frac{1}{n}\sum_{i=1}^{n} \dfrac{2(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{\begin{pmatrix}((T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2) \\ +(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2\end{pmatrix}}$ and $S_C(A,B) = \frac{1}{n}\sum_{i=1}^{n} \dfrac{T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)}{\sqrt{(T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2}\sqrt{(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2}}$ are defined by Ye (2014).
- $S_{Cot1}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \cot\left[\frac{\pi}{4} + \frac{\pi}{4}\max\begin{pmatrix}|T_A(x_i) - T_B(x_i)|, \\ |I_A(x_i) - I_B(x_i)|, \\ |F_A(x_i) - F_B(x_i)|\end{pmatrix}\right]$ and $S_{Cot2}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \cot\left[\frac{\pi}{4} + \frac{\pi}{12}\max\begin{pmatrix}|T_A(x_i) - T_B(x_i)| \\ + |I_A(x_i) - I_B(x_i)| \\ + |F_A(x_i) - F_B(x_i)|\end{pmatrix}\right]$ are defined by Ye (2015).

To review the performance of similarity measures let us consider an example. Consider the following four SvNSs

$A_1 = \{\langle x, 0.1,0.2,0.2\rangle\}$, $A_2 = \{\langle x,0.2,0.4,0.4\rangle\}$, $A_3 = \{\langle x,0.1,0.2,0.3\rangle\}$ and $A_4 = \{\langle x,0.0,0.0,0.0\rangle\}$.

Table 1 shows the comparison of proposed measures with some existing measures of similarity between two SvNSs.

TABLE 1 Comparison between similarity measures

	S_{BS}	S_{BS1}	S_J	S_D	S_C	S_{Cot}	S_{Cot1}	S_w	S
A_1A_2	0.866667	0.5	0.66666	0.8	1	0.7265	0.7673	0.833333	1
A_3A_4	0.8	0	0	0	Not Defined	0.6128	0.7265	0.75	1

It is clear from Table1 that S_C and S are unresonable in determining similarity between A_1, A_2, and A_3, A_4. Where as S_{BS1}, S_J and S_D are zero for A_3A_4. Thus S_{BS}, S_{cot}, S_{cot1} and S_w are able to determine similarity in a better way.

CONCLUSION

Distance and similarity measure are significant research area in neutrosophic information theory as they are efficient tools to deal with uncertain and insufficient information. In this paper we have derived averaged distance measure and derived similarity measure using the proposed distance measure and the relation between distance and similarity measure under single valued neutrosophic environment. Further, another similarity measure is proposed for SvNSs. Next, a method is derived to define ideal /optimal point using the existing information as previous literature used $\langle 1,0,0 \rangle$ as optimal point which does not exists in reality. Further, proposed similarity measures along with defined optimal point are used to solve multi-attribute decision making problem, which helps in providing rank to all the available alternatives and helps in identifying the best one. An illustrative example is used to demonstrate its application. Finally the proposed similarity measures are compared with some existing similarity measures. Proposed similarity measure S_w is proved to be better than some existing measures.

BIBLIOGRAPHY

I. Atanassov, K. (1999). Intuitionistic Fuzzy Sets. Heidelberg, New York: Physica-Verlag.

ii. Aydoğdu, A. (2015). On Similarity and Entropy of Single Valued Neutrosophic Sets. Gen. Math. Notes, 29(1), 67-74.

iii. Broumi, S., & Smarandache, F. (2013). Several Similarity Measures of Neutrosophic Sets. Neutrosophic Sets and Systems,, I, 54-62.

iv. Grzegorzewski, P. (2004). Distance Between Intuitionistic Fuzzy Sets and/or Interval-valued Fuzzy Sets Based in the Hausdorff Metric. Fuzzy Sets and Systems, 48, 319-328.

v. Kacprzyk. (1997). Multistage Fuzzy Control. Chichester: Wiley.

vi. Park, J. H., Lin, K. M., Park, J. S., & Kwun, Y. C. (2007). Distance between Interval-valued Intuitionistic Fuzzy Sets. Journal of Physics: Conference Series, 96.

vii. Smarandache, F. (1998). A Unifying Field in Logics. Neutrosophy: NeutrosophicProbability, Set and Logic. Rehoboth: American Research Press.

viii. Smarandache, F. (2005). Neutrosophic Set, a Generalisation of the Intuitionistic Fuzzy Sets. International Journal of Pure Applied Mathematics, 24, 287-297.

ix. Szmidt, E., & Kacprzyk, J. (2000). Distance between Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 114(3), 505-518.

x. Wang, H., Zhang, Y., Sunderraman, R., & Smarandache, F. (2011). Single Valued Neutrosophic Sets. Fuzzy Sets, Rough Sets and Multivalued Operations and Applications,, 3(1), 33–39.

xi. Yang, Y., & Chiclana, F. (2012). Consistency of 2D and 3D Distance of Intuitionistic Fuzzy Sets. Expert Systems with Applications, 39(10), 8665-8670.

xii. Ye, J. (2014). Clustering Methods Using Distance-Based Similarity Measures of Single-Valued Neutrosophic Sets. Journal of Intelligent Systems, 379-389.

xiii. Ye, J. (2014). Similarity Measures between Interval Neutrosophic Sets and their Multicriteria Decision-Making Method. Journal of Intelligent and Fuzzy Systems, 26, 165-172.

xiv. Ye, J. (2015). Improved Cosine Similarity Measures of Simplified Neutrosophic Sets for Medical Diagnoses. Artificial Intelligence in Medicine, 63, 171-179.

xv. Ye, J. (2015). Improved Cross Entropy Measures of Single Valued Neutrosophic Sets and Interval Neutrosophic Sets and Their Multicriteria Decision Making Methods. Cybernetics and Information Technologies, 15(4), 13-26.

xvi. Ye, J., & Smarandache, F. (2016). Similarity Measure of Refined Single-Valued Neutrosophic Sets and Its Multicriteria Decision Making Method. Neutrosophic Sets and Systems, 12, 41-45.